Evaluation of form Tolerance for Cylindricity

Swathi.M Bale, P. Dinesh, L. Gopinath and S. Ravishankar

Abstract— Evaluation of form error is a critical aspect of many manufacturing process because this tolerance requires verification in all three dimensions. Many methods have been proposed in efforts to improve upon the results obtained from co –ordinate measuring machine. In this work, a least square technique is introduced for use in evaluating the forms of straightness, flatness, circularity, cylindricity and perpendicularity. The part considered for the work is torque shaft. The shaft is divided into segments at different z values for the evaluation. The circularity is calculated by considering the two aspects, mean radius and also the centre of the circle. The mathematical evaluation code is generated for Least Square method and then solved using software program written in C language. The input data is taken by inspecting the torque shaft by advanced measuring instrument, co-ordinate measuring machine. For all the forms considered the proposed new formulations are found to be equal or better than the co ordinate measuring machine (CMM) values. The results obtained from the developed code show better results as compared with the values of CMM.

Keywords--- Form Tolerance, Cylindricity, Least Square Method

I. INTRODUCTION

TOLERANCE is defined as the magnitude of permissible variation of a dimension or other measured or control criterion from specified value. Tolerances have to be allowed because of the inevitable human failings and machine limitations which prevent ideal achievement during fabrication. The primary purpose of tolerance is to permit variation in dimension without degradation of the performance is the criterion; there the functional requirements will be the dominating factor in setting tolerances. Evaluation of form tolerance plays a very important role in many manufacturing industries for the specified tolerance verification. In many manufacturing industries inspite of using high range technologies there are more chances that the manufactured part may reach to its final required shape but with some deviation.

Advanced manufacturing processes are induced in the manufacturing domain to achieve precision and exact parts meeting the requirements with application. Tolerance is a feature to accommodate the variations subject to the optimization of exactness against variables. Form tolerance helps to monitor the parts, exactness to the design and model. The form tolerance absolves the geometries like flatness, straightness, circularity, cylindricity. Evaluation of form tolerance is a critical aspect of many manufacturing process. Form tolerance is defined as a group of geometric tolerances that limit the amount of error in the shape of a feature. A great majority of mechanical parts comprise of cylindrical features are considered for cylindricity such as push pull tube, inner and outer cylinder, wheel axle, cylindrical crankpin, piston which are used in aircraft. The component considered in this study is torque shaft of an aerospace application. The component is measured using the CMM.

Cylindricity is a condition of a surface revolution in which all the points of the surface are equidistant from a common axis. A cylindricity tolerance specifies a tolerance zone bounded by two concentric cylinders within which the surface must lie. In Geometric Dimensioning and Tolerancing, cylindricity tolerance is used when cylindrical part features must have good circularity and straightness, like pins or camshafts. While circularity applies only to cross sections, cylindricity applies simultaneously to the entire surface. Since cylindricity is applied to an individual surface, this tolerance does not need to be related to a datum.

A common reason for cylindricity control to be used on a drawing is to limit the surface conditions (out of round, straightness and taper) of a shaft diameter. The cylindricity control limits the maximum allowable cylindricity error.

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PAPER ID: MED23
Conditions for cylindricity control are: The cylindricity control must be within its size tolerance, the cylindricity control tolerance must be less than the total size tolerance, cylindricity control does not affect the outer boundary of the feature of size.

In the area of form tolerance measurement many techniques have been suggested and implemented over the years [2-10]. Normal Least Square method is one among them and used in present study which minimizes the sum of the squares of the perpendicular distances to the axis of the component from different points [1].

1.1 Specifications of Test for Cylindricity control as per ANSI Y 14.5 M are:

For cylindricity control to be a legal specification [12], it must satisfy the following conditions:
1. No datum can be specified in feature control frame.
2. No modifiers can be specified in the feature control surface.
3. The control must be applied to a cylindrical feature.
4. The cylindricity control tolerance value must be less than any geometric control that limits the cylindricity of the feature.

II. PROBLEM DEFINITION

A shaft is an element used to transmit power and torque, and it can allow reverse bending. Most shafts have circular cross sections, either solid or tubular.

Least square approximation is the mathematical procedure for finding the best-fitting curve to a given set of points by minimizing the sum of the squares of the offsets ("the residuals") of the points from the curve. The method of least squares is a standard approach to the approximate solution of over determined systems, i.e., sets of equations in which there are more equations than unknowns. "Least squares" means that the overall solution minimizes the sum of the squares of the errors made in solving every single equation. The sum of the squares of the offsets is used instead of the offset absolute values because this allows the residuals to be treated as a continuous differentiable quantity. The goal of Least-Squares Method is to find a good estimation of parameters that fit a function, f(x), of a set of data, x₁...xₙ. The Least-Squares Method requires that the estimated function has to deviate as little as possible from f(x) in the sense of L2-norm.

Cylindricity one among the form tolerances has an important role in industries because of the central role of bearings and shafts in machines. Cylindricity is more complex than straightness, flatness, or circularity, not because it is three-dimensional, but also because it is defined by a mixture of Cartesian and polar coordinate systems. Cylindricity is a condition on a surface of revolution in which all points of the surface are equidistant from a common axis. Significant error associated with this characteristic may result in the failure or inadequate functioning of the corresponding part. Accurate measurement of this error is not a trivial task due to the 3D nature of the characteristic.

Evaluation of cylindricity considers following features:

i. Circularity of cylinder.
ii. Straightness of cylinder.
iii. Parallelity of cylinder planes.
iv. Perpendicularity of cylinder axis with planes.

III. METHODOLOGY

3.1 Experimental Details

A huge number of mechanical parts embrace of cylindrical features such as shafts and revolving devices. A vital geometric characteristic that is used to control form and function of cylindrical features is cylindricity. The part considered for cylindricity tolerance is the torque shaft figure 1. The part is measured by advanced electronic measuring system such as Coordinate Measuring Machine (CMM) available at NAL, Bangalore. The diameter of the probe used for the measurement of the torque shaft is 5 mm. The total length of the part is 83 mm with the diameters 47 mm, 40mm and 39mm for the height of 13mm, 30mm, and 40mm respectively. The part is placed vertically as shown in figure 1. To fix the datum along the z axis of the cylinder the flatness for the plane at the top and bottom of cylinder is measured and the planes have a flatness of 63 microns. The shaft is measured at different z (6mm, 12mm, 20mm, 30mm, 40mm, 50mm, 60mm, 70mm, 80mm) levels for circularity along the circumference. For circularity measurement the centre of the circle (x, y) at a distance of 41.5mm is set as zero, this is done by measuring the co
ordinates along the circumference of the circle at minimum of six points, with the reference of this point the co ordinates along the circumference is measured for circles at different z levels.

3.2 Mathematical Evaluation

The following paragraphs detail method of Least Square used in present work.

- **Least Square Best Fit Circle:**

  The procedure to fit a circle of m data points \((x_i, y_i)\), where \(m \geq 3\), is given as follows. The general equation of the circle is given by,

  \[
  (x-x_0)^2 + (y-y_0)^2 = R^2 \tag{1}
  \]

  The least square function for the circle equation is given as,

  \[
  S = \sum_{i=1}^{m} [g(u,v)]^2 \tag{2}
  \]

  Where,

  \[
  g(u,v) = (u-u_c)^2 + (v-v_c)^2 - \alpha \tag{3}
  \]

  \[
  u = \bar{X}_c - x \\
  v = \bar{Y}_c - y \\
  \bar{X}_c = \frac{\sum X_i}{m} \\
  \bar{Y}_c = \frac{\sum Y_i}{m} \\
  (u_c, v_c) = c \\
  \alpha = R^2
  \]

  \(c\)- center of the circle

  In equation (3) the unknown variables are \(u_c, v_c\) and \(\alpha\). Hence finding the partial derivatives of equation (3) with respect to the variables independently and solving the resultant will give the values of \(u_c, v_c\) and \(\alpha\) which can be summarized as follows:

  \[
  u_c = \frac{1}{2} \frac{\sum uv (\sum v^2 + \sum u(v)^2) - \frac{1}{2} [v^2 (\sum u^3 + \sum u(v)^2)]}{(\sum uv)^2 - (\sum u)^2 (\sum v)^2} \\
  v_c = \frac{1}{2} \frac{\sum uv (\sum u^2 + \sum u(v)^2) - \frac{1}{2} [u^2 (\sum v^3 + \sum v(u)^2)]}{(\sum uv)^2 - (\sum u)^2 (\sum v)^2} \\
  \alpha = \frac{u^2 + v^2 + \sum u^2 + \sum v^2}{m}
  \]

  the centre and the radius of the required circle are given as follows :

  \[
  R = \sqrt{\alpha} \\
  \text{Center } (x_c, y_c) = (u_c, v_c) + (\bar{x}, \bar{y})
  \]

  In the present work the following mathematical calculations are followed in the evaluation of cylindricity:
The possible deviations for the circularity occur in two ways:

- With the centre being the ideal point \((x_o=0, y_o=0)\), the difference of radii \((r_c-r_o)\) i.e., designers requirement \(r_i\) figure 2 manufactured radius \(r_o\) figure 3 is the actual deviation of the part with known co ordinates of the centre fixed as a reference point.

- Cylinders are manufactured in the focus of maintaining the circularity and its envelop all along the datum. Datum of reference centre is usually located and aligned during manufacturing (like in lathe chuck centre alignment is referenced to zero before metal cutting). Inspection of these surfaces for circularity is an added task: to manipulate a centre and calculate the equidistant value from the manipulated centre.

The deviation of radius \((r_c-r_o)\) presents the deviation in circularity with respect to manipulated centre, \(r_c\) is the distance between manipulated centre and the manufactured circumference, \(r_o\) is the distance between ideal centre and the manufactured circumference.

The deviation of radius \((r_c-r_i)\) evaluated by manipulated centre \((x_c, y_c)\) and the ideal reference centre \((x_o, y_o)\) gives the cumulative variation because of mathematically evaluated centre and the manufactured circumference.

- The chance in circumference only because the centre is shifted. Best fit of this is attempted by fitting the circle by Least Square Method (LSM).

- **Straightness:**
  
  Equation of straight line,
  
  \[ Y = bx + c \]

  Applying least square method,

  \[ \sum y = b \sum x + nc \]

  \[ \sum xy = b \sum x^2 + c \sum x \]

  Where,

  \( b \) is the slope of the line.
  \( c \) is y intercept.
  \( n \) is the numbers of points considered.

  Solving the above equations the value of \( x \) and \( y \) are calculated. The above equations are used to calculate, how far the centers of the circle are deviated from the original centre.

- **Plane:**
  
  Equation of the plane is \( Ax + By + Cz + D = 0 \)

  \[
  \begin{bmatrix}
  x - x_1 & y - y_1 & z - z_1 \\
  x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\
  x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\
  \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = 0
  \]

  The above matrix is solved to get the equation of the plane.

  Where,

  \( A, B, C, D \) are direction cosines of the plane.

- **Perpendicularity of the Plane:**
  
  Equation of the plane and the line is,

  \( Ax + By + Cz + D = 0 \)
Y=bx+c

From the equation of the plane, the coefficients yield the components of the normal vector (which is perpendicular to the plane and hence can be used in the equation of the desired line).

\[ n = A \mathbf{i} + B \mathbf{j} + C \mathbf{k} \]

Hence the parametric equations for line involve the given point and this vector, becoming

\[ x = x_1 + At, \quad y = y_1 + Bt, \quad z = z_1 + Ct. \]

where,

\[ x_1, \quad y_1, \quad z_1 \]

are the points through which the line is passing.

The symmetric equations for line can be found by solving this last equation for t.

\[ t = \frac{x-x_1}{A} = \frac{y-y_1}{B} = \frac{z-z_1}{C} \]

For the x-y plane, \( z = 0 = z_1 + Ct_{xy} \) to find the value of the parameter t for the x-y plane:

\[ t_{xy} = \frac{z_1}{c} \]

This value of t is used in the equations for calculating x and y: \( x_{xy} = x_1 + At_{xy} = x_1 + A(z_1/c) \). A program written in C language is used to solve each of the formulations.

IV. RESULTS AND DISCUSSIONS

The results obtained with and Least Square method (LS) for circularity at each z level is tabulated table1 and the values are compared with the values obtained with power inspect software. The graph for L1 norm and the Least Square method are plotted at z levels of 20mm, 40mm, 50mm and the normal distribution curve for the Least Square method at same z levels is plotted. The straightness of the centers of the circle at all z levels is calculated. Perpendicularity of plane with axis of the cylinder with the line passing through the centre of the cylinder is calculated.

Straightness tolerance using least square method considering all z levels: 0.035821 millimetre. Perpendicularity of plane with the line passing through the centre of the cylinder: 0.1404 degree.

Table 1: Calculated Circularity at Different z levels with L1, L2 Norm Compared with Power Inspect Software Values.

<table>
<thead>
<tr>
<th>Z levels (mm)</th>
<th>L1 (mm)</th>
<th>PI (mm)</th>
<th>LS (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(ro-rc)</td>
<td>(ro-avg ro)</td>
<td>(ro-avg rc)</td>
</tr>
<tr>
<td>Z=6</td>
<td>0.03139</td>
<td>0.0089396</td>
<td>0.0086918</td>
</tr>
<tr>
<td>Z=12</td>
<td>0.02873</td>
<td>0.006964</td>
<td>0.0307248</td>
</tr>
<tr>
<td>Z=20</td>
<td>0.029815</td>
<td>0.019053</td>
<td>0.0278764</td>
</tr>
<tr>
<td>Z=30</td>
<td>0.873256</td>
<td>0.8975189</td>
<td>0.0283447</td>
</tr>
<tr>
<td>Z=40</td>
<td>0.82388</td>
<td>0.00567958</td>
<td>0.027166</td>
</tr>
<tr>
<td>Z=50</td>
<td>0.828185</td>
<td>0.00592657</td>
<td>0.0263857</td>
</tr>
<tr>
<td>Z=60</td>
<td>0.192844</td>
<td>0.171377</td>
<td>0.0263085</td>
</tr>
<tr>
<td>Z=70</td>
<td>0.021622</td>
<td>0.00608568</td>
<td>0.0257014</td>
</tr>
<tr>
<td>Z=80</td>
<td>0.021445</td>
<td>0.00566808</td>
<td>0.0250916</td>
</tr>
</tbody>
</table>

Figure 4: Graph Representing (Circularity) L2 considering the Centers and the Radius for Cylinder Points at z=20mm.
The graphs, figures 4 and the deviations in the error for different methods proposed in this study and the normal distribution curve for the same L2 values are plotted, figures 5.

V. CONCLUSION

From this study, it is evident that the Least Square (with known and calculated centre) approximation technique is a viable method for form tolerance evaluation. The graphs plotted for Least square and L1 approximation over the suggested (with known and calculated centre) method have a smooth curve when compared to other methods of evaluation which shows that the error is uniformly distributed along the circumference of the cylinder. Hence the method suggested is best suited for the evaluation of form tolerance.

REFERENCES