A Novel Method for the Design of LQR Controllers in Speed Control of DC Motor

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Abstract--- *In this paper, a novel approach is presented to take care for the design of linear quadratic regulator (LQR) problem for the selection of Q and R matrices in the theory of optimal control for the speed control of DC motor. Detailed Investigations have been carried out to a general second order system and the characteristic equation of the controlled system to make the necessary inferences that include the properties: damping ratio (ζ) and natural frequency (ω_n). The novelty of the approach is that the selection of the elements of the performance index matrices Q and R is not carried out by trial and error method as in earlier conventional LQR controllers, but by using relevant mathematical relations that will specify the system's steady and transient response. The proposed novel method presented here is tested on the MATLAB /SIMULINK® platform. A comparison of the step response of DC motor between the conventional LQR method (trial and error method for the selection of Q and R matrices) and the proposed novel method with respect to prominent parameters such as armature current and rotor angular velocity is presented.*

Index Terms--- *LQR, DC Model, Trial & Error, Ricatti Equation*

I. Introduction

DC motors provide an attractive alternative to AC servomotors in high-performance motion control applications [1]. Due to the excellent speed control characteristics of a DC motor, it has been widely used in industry (such as cars, trucks and aircraft) even though its maintenance costs are higher than the induction motor.

As a result, researchers have paid attention to position control of DC motor and prepared several methods to control speed of such motors. In recent years several publications propose alternative approaches to identification and control of DC motors. Umeno and Hori (1991) describe a generalized speed control design technique of DC servomotors based on the parameterization of two-degrees-of-freedom controllers and apply the design method of a Butterworth filter to determine the controller parameter. Chevrel et al. (1996) present a switched LQR speed controller, designed from the linear model of the DC motor, and compare its performance with a cascade control design in terms of accuracy, robustness and complexity. Rubaai and Kotaru (2000) propose an alternative way to identify and control DC motors by means of a nonlinear control law represented by an artificial neural network. [Gwo, 2004], presented a novel optimal PID controller using (LQR) methodology in tuning the parameters of PID controller. Yu and Hwang (2004) [2] present an LQR approach to determine the optimal PID speed control of the DC motor. Ruba M. K, Al-Mulla Hummadi. (2012) describes DC motor speed control based on optimal LQR technique [3]: Controller objective of this paper is to maintain the speed rotation of the motor shaft with a particular step response.

In particular, if we restrict our attention to LQR, it can be argued that the basic principles for implementing such regulators are choosing the elements of the weighting matrices Q and R, which effect states and control respectively, is left to the controller. Every different value of Q and R will eventually end up with a different system response even if all of these control strategies are optimal by nature. Previous attempts to model such systems from LQR theory [3], [4], [5], the two matrices Q and R are selected by design engineer by trial and error method which needs to be a large time domain design.

The aim of this research is to develop relevant mathematical relations for the existing DC motor model based on second order system parameters ζ and $\omega_n$to select the Q and R weighting matrices which would overcome the time domain design and improves output performance compared to the earlier conventional LQR methods [3],[4].

The remainder of the paper is organized as follows. Section 2 describes the system configuration and modeling of a DC motor. The design of the LQR control algorithm is detailed in Section 3. Section 4 describes the proposed novel method for the selection of Q and R matrices. Results and analysis follow in the concluding section.

II. DC Motor Model

A simple motor model is shown in Fig.1. The armature circuit consist of a resistance (R_a) connected in series with an inductance (L_a), and a voltage source (e_b) representing the back emf (back electromotive force) induced in the armature when during rotation. [Ogata, 1998 and 2002].

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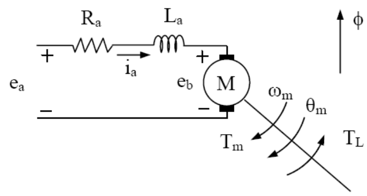

Figure 1: DC-Motor Model

From Fig. 1 we can write the following equations based on the Newton's law combined with the Kirchhoff's law:

$$\frac{di_a}{dt} = \frac{R}{L} i_a - \frac{K}{L}\omega_m + \frac{1}{L} V_a \quad (1)$$

$$\frac{d\omega_m}{dt} = \frac{K}{J} i_a - \frac{B_m}{J}\omega_m \quad (2)$$

In the state-space form, the equations above can be expressed by choosing the rotating speed and electrical current as the states variables and the voltage as an input.

$$\begin{bmatrix} \frac{di_a}{dt} \\ \frac{d\omega_m}{dt} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & -\frac{K}{L} \\ \frac{K}{J} & -\frac{B_m}{J} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} V_a = Ax + Bu \quad (3)$$

$$y = [0 \quad 1] \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + [0]V_a = Cx + Du \quad (4)$$

All the relevant DC motor system parameters and variables along with their values used in the experiment are described in the appendix section at the end of paper.

III. LQR Control Algorithm

One of the MIMO design approach is the optimal control method of linear quadratic regulator (LQR). The idea is to transfer the designer's iteration on pole locations as used in full state feedback to iterations on the elements in a cost function, J. This method determines the feedback gain matrix that minimizes J in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system [6], [7].

For a given system

$$\dot{X} = Ax(t) + Bu(t)$$

Determine the matrix K of the LQR vector

$$u(t) = -KX(t)$$

In order to minimize the performance index,

$$J = \frac{1}{2}\int_0^{\infty} (x^T Q x + uRu)dt$$

Where Q and R are the positive-definite Hermitian or real symmetric matrixes, the matrix Q and R determine the relative importance of the error and the expenditure of this energy. From the above Equations

$$K = -R^{-1}B^T P$$

And hence the control law is

$$u(t) = -KX(t) = -R^{-1}B^T P X(t)$$

In which P must satisfy the reduced Riccati Equation:

$$PA + A^T P - PBR^{-1} + B^T P + Q = 0. \quad (5)$$

The LQR design selects the weight matrix Q and R such that the performances of the closed loop system can satisfy the desired requirements mentioned earlier. The selection of Q and R is weakly connected to the performance specifications, and a certain amount of trial and error is required with an interactive computer simulation before a satisfactory design results.

IV. Proposed Novel Method for the Selection of Q and R Matrices

For any second order systems, the matrices Q, R and P can be chosen as

$$Q = \begin{bmatrix} q_1 & 0 \\ 0 & q_2 \end{bmatrix} \qquad R = r \qquad P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$$

State space matrices A, B, C and D can be considered in the controllability canonical form as

$$A = \begin{bmatrix} 0 & 1 \\ A_{21} & A_{22} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ B_{21} \end{bmatrix} \qquad C = [b_2 \quad b_1] \qquad D = 0$$

The control gain is given by

$$K = R^{-1}B^T P = \frac{B_{21}}{r}[P_2 \quad P_3] \quad (6)$$

Here, P_2 and P_3 are obtained from the solution of the Riccati Equation (5) as

$$P_2A_{21} + P_2A_{21} - \frac{P_2^2 B_{21}^2}{r} + q_1 = 0 \quad (7)$$

$$P_3A_{21} + P_1 + P_2A_{22} - \frac{P_2P_3B_{21}^2}{r} = 0 \quad (8)$$

$$P_1 + P_2A_{22} + P_3A_{21} - \frac{P_2P_3^2B_{21}^2}{r} = 0 \quad (9)$$

$$P_2 + A_{22}P_3 + P_2 + P_3A_{22} - \frac{P_3^2B_{21}^2}{r} + q_2 = 0 \quad (10)$$

Solving (7) and (10) P_2 and P_3 are obtained as

$$P_2 = \frac{A_{21}}{B_{21}^2} r + \frac{1}{B_{21}^2}\sqrt{(A_{21}r)^2 + B_{21}^2 q_1 r} \quad (11)$$

$$P_3 = \frac{A_{22}}{B_{21}^2} r + \frac{1}{B_{21}^2}\sqrt{(A_{22}r)^2 + (2P_2 + q_2)B_{21}^2 r} \quad (12)$$

A. *Response of the Controlled System*

The optimal states of the controlled plant can be found by solving the first order differential equation obtained by using the optimal control (6) in the plant (3).

$$\dot{X} = [A - BK]X(t) = [A - BR^{-1}B^T P]X(t) \quad (13)$$

For a closed-loop optimal system like (13) to be stable; the matrix $[A - BK]$ should have stable Eigen values, which also means that they should have negative real parts. To analyze the response of the system, characteristic equation is obtained as

$$\det(A - Bk) = 0$$

That yields,

$$s^2 + \left(\frac{B_{21}^2}{r}P_3 - A_{22}\right)s + \left(\frac{B_{21}^2}{r}P_2 - A_{21}\right) = 0 \quad (14)$$

Equation (14) is in the form of $s^2 + 2\xi\omega_n s + \omega_n^2 = 0$that isthe general form of the characteristic equation for any second order system. Hence, damping ratio and natural frequency of the controlled system are found as

$$\omega_n^2 = \left(\frac{B_{21}^2}{r}P_2 - A_{21}\right) \quad 2\xi\omega_n = \left(\frac{B_{21}^2}{r}P_3 - A_{22}\right)$$

Substituting the value of $\mathbf{P_2}$ and $\mathbf{P_3}$ from (11) & (12)

$$\omega_n = \left(\frac{B_{21}^2}{r} * \left(\frac{A_{21}}{B_{21}^2} r + \frac{1}{B_{21}^2}\sqrt{(A_{21}r)^2 + B_{21}^2 q_1 r}\right) - A_{21}\right)^{\frac{1}{2}}$$

$$\omega_n = \left[\left(A_{21} + \frac{1}{r}\sqrt{(A_{21}r)^2 + B_{21}^2 q_1 r}\right) - A_{21}\right]^{\frac{1}{2}}$$

$$\omega_n^2 = \left[A_{21}^2 + B_{21}^2\left(\frac{q_1}{r}\right)\right]^{\frac{1}{2}}$$

$$\omega_n = \left[A_{21}^2 + B_{21}^2\left(\frac{q_1}{r}\right)\right]^{\frac{1}{4}} \qquad (15)$$

$$2\xi\omega_n = \left[A_{22}^2 + B_{21}^2\left(\frac{q_2}{r}\right) + 2A_{21} + 2\sqrt{A_{21}^2 + B_{21}^2\left(\frac{q_1}{r}\right)}\right]^{\frac{1}{2}}$$

$$\xi = \frac{1}{2\omega_n}\left[A_{22}^2 + B_{21}^2\left(\frac{q_2}{r}\right) + 2A_{21} + 2\sqrt{A_{21}^2 + B_{21}^2\left(\frac{q_1}{r}\right)}\right]^{\frac{1}{2}} \qquad (16)$$

In equations (15) and (16), system characteristics are rewritten in the terms of the elements of Q and R. Therefore, the desired conditions are somewhat transformed into the task of the Q and R selection

To summarize, the control designer should determine the ratios $\left(\frac{q_1}{r}\right)$and $\left(\frac{q_2}{r}\right)$to obtain the desired optimal behavior of the system. Individual values of these parameters are not important in this task; whereas their ratios are to be considered. The control philosophy turns out as:

1. Specify overshoot value and steady state time as constraints.
2. Calculate damping ratio from overshoot value and
3. natural frequency from steady state time.
4. Solve (15) for $\left(\frac{q_1}{r}\right)$and (16) for $\left(\frac{q_2}{r}\right)$.
5. Set r value free of any constraint, obtain$\left(\frac{q_1}{r}\right)\left(\frac{q_2}{r}\right)$.

V. SIMULATION RESULTS

The experimental set-up to test the proposed method consists of A and B matrices (controller canonical form) are given below.

$$A = \begin{bmatrix} 0 & 1 \\ -27{,}630 & -675.1 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C = [262500 \quad 250]$$

The earlier conventional LQR [2], [3] controller gain K can be obtained by selecting weighting matrices in trial and error method. In order to improve the design, the various Q matrices and R values were tried. In one of the LQR method [3] the Q and R matrices are chosen as

$$Q = \begin{bmatrix} 1 & 0 \\ 0 & 10 \end{bmatrix} \qquad R=0.01$$

$$K = [0.0018 \qquad 0.7402]$$

The proposed novel method controller gain K can be obtained by solving Equations (14) & (15) for the Q and R matrices based on the second order system parameter values ζ=1 and ω_n=350 are

$$Q = \begin{bmatrix} 142.42e+06 & 0 \\ 0 & 46.49 \end{bmatrix} \qquad R=0.01$$

$$K = [9.4867e+04 \qquad 0.0131e+04]$$

The matrix C is vector with zeros along with constant value in any one position depending on the state variable on which the performance index is based.

The dynamic response curves for the state variables Armature current and Rotor angular velocity of the DC motor are plotted as shown in the Fig's 2-3 with the legend trial and error method and proposed novel method for the control of DC motor.

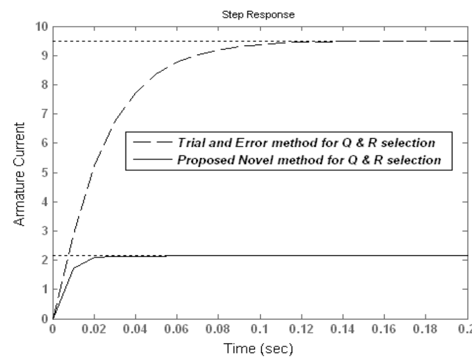

Figure 2: Conventional LQR Step Response of Armature Current

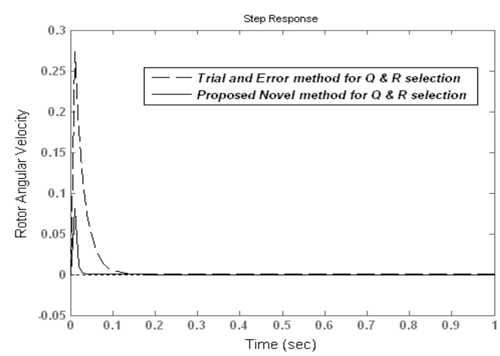

Figure 3: Conventional LQR Step Response of Rotor Angular Velocity

Table 1: Comparison of Simulation Results

State Variables	LQR control by Trial and Error method for the selection Q & R		
System Parameters	Peak Amplitude	Settling Time	Peak Overshoot
Armature Current	9.5	0.15 sec	0%
Rotor Angular Velocity	0.28	0.12 sec	27%
State Variables	LQR control by Proposed Novel method for the selection Q & R		
System Parameters	Peak Amplitude	Settling Time	Peak Overshoot
Armature Current	2	0.02 sec	0%
Rotor Angular Velocity	0.08	0.01 sec	6%

VI. Conclusion

Speed control of a DC motor is an important issue, so this paper presents a novel design method for the selection of Q and R matrices to determine the optimal speed control using LQR method. From Figures 2-3 and Table one we can conclude that the proposed novel method for the selection of Q & R based LQR controller provides better performance (shorter settling time & smaller overshoot) compared to the performance obtained from the earlier Convectional LQR control.

VII. Appendix

A = n×n constant matrix

B = n×1 constant matrix

B_m = viscous friction coefficient (kgm2/s) = 0.0000093.

C =1×n constant matrix

$V_a(t)$ = applied voltage (V)

$i_a(t)$ = armature current (A)

J = moment of inertia of rotor (kg.m2) = 0.0001.

K = torque constant (Nm/A) = 0.105.

L = armature inductance (H) = 0.004.

R = armature resistance (Ω) = 2.7.

ω_m (t)= rotor angular velocity (rad/s).

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