Synchronization of n-scroll Chua Circuit via Adaptive Control Design Based on Feedback Control

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Abstract

This paper investigates the adaptive control design with feedback input approach for controlling chaotic systems to ensure global chaos synchronization of chaotic systems, viz. n–scroll Chua circuit. Our theorem on synchronization for n–scroll Chua circuit is established using Lyapunov stability theory. The adaptive control links the choice of a Lyapunov function with the design of a controller and guarantees global stabilities performance of strict-feedback nonlinear systems. The adaptive control method is effective and convenient to synchronize and estimate the parameters of the chaotic systems mainly this technique gives that the flexibility to construct a control law and estimate the parameter values. Numerical simulations are also given to illustrate and validate the synchronization results derived in this paper.

Keywords: Chaos, Synchronization, Adaptive Control, Chua circuit.
2000 MSC: (34H10, 93C15, 34H15)

1. Introduction

Chaos refers to one type of complex dynamical behaviors that possess extreme sensitivity to tiny variations of initial conditions, bounded trajectories in phase space and fractional topological dimensions. Synchronization research has been focused on the state observers, where the main applications pertain to the synchronization of nonlinear oscillators and the use of control laws, which allows to achieve the synchronization between nonlinear oscillators, with different structures and orders.

The synchronization of chaotic system was first researched by Yamada and Fujisaka [1] with subsequent work by Pecora and Carroll [2, 3]. The synchronization of chaos is one way of explaining sensitive dependence on initial conditions. It has been established that the synchronization of two chaotic systems, that identify the tendency of two or more systems are coupled together to undergo closely related motions. The problem of chaos synchronization is to design a coupling between the two systems such that the chaotic time evaluation becomes ideal. The output of the slave system asymptotically follows the output of the master system i.e. the output of the master system controls the slave system.

The synchronization for chaotic systems has been widespread to the scope, such as generalized synchronization [4], phase synchronization [5], lag synchronization, projective synchronization [6], generalized projective synchronization [7, 8] and even anti-synchronization. A variety of schemes for ensuring the control and synchronization of such systems have been demonstrated based on their potential applications in various fields including chaos generator design, secure communication [9, 10], physical systems [11], and chemical reaction [12], ecological systems [13], information science [14], energy resource systems [15], ghostburster neurons [16], bi-axial magnet models [17], neuronal models [18, 19], IR epidemic models with impulsive vaccination [20] and predicting the influence of solar wind to celestial bodies [21], etc. So far a variety of impressive approaches have been proposed for the synchronization of the chaotic systems such as the OGY method[22], sampled feedback synchronization method, time delay feedback method [23], adaptive design method [24–26], sliding mode control method [27], active control method [28], backstepping control [29, 30] etc.

Adaptive control design is a direct aggregation of a control methodology with some form of a recursive system identification and the system identification could be aimed to determining the system to be controlled is linear or

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nonlinear systems. The system identification is only the parameters of a fixed type of model that need to be determined and limiting the parametric system identification and parametric adaptive control. Adaptive control design is studied and analyzed in theory of unknown but fixed parameter systems.

In this paper, Adaptive control design with feedback input approach is proposed. This approach is a systematic design approach and guarantees global stability of the n-scroll Chua chaotic circuit. Based on the Lyapunov function, the adaptive update control is determined to tune the controller gain based on the precalculated feedback control inputs. We organize this paper as follows. In Section 2, we present the methodology of chaos synchronization by adaptive control method. In Section 3, we give a description of the chaotic systems discussed in this paper. In Section 4, we demonstrate the chaos synchronization of identical n–scroll Chua systems [31]. In Section 5, we summarize the results obtained in this paper.

2. Problem Statement and Our Methodology

In general, the two dynamic systems in synchronization are called the master and slave system respectively.

Consider the dynamics of nonlinear systems whose trajectories are having chaotic attractor

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is a state vectors of the system.A is the $n \times n$ matrix of the system parameters and $f : \mathbb{R}^n \to \mathbb{R}^n$ is the nonlinear part of the system. Assume that the master system is described as eqn(1) and the slave system which is coupling to eqn(1). The adaptive slave systems described so far are the special form

$$\dot{y} = Ay + f(y) + u \tag{2}$$

where u is the input to the system with parameter estimator $\hat{\alpha}_i, i = 1, 2, 3, ..., n$ and $y(t) \in \mathbb{R}^n$ is a over all state vectors of the system including the controller and identifier. If f equals to g, then the systems states are identical synchronization otherwise that systems states are non identical chaotic synchronization of systems. The chaotic systems (1) and (2) depends not only on state variables but also on time t and the parameters. The problem is to analyze the synchronization between two chaotic systems are transform to another problem on how to choose the control law $u_i, i = 1, 2, 3, ..., n$ and the parameter identifier $\hat{\alpha}_i, i = 1, 2, 3, ..., n$ to make the e converge to zero with the time increasing.

In order to observe the synchronization behavior in master and slave systems, we have introduced the control functions $u_i, i = 1, 2, 3, ..., n$ and the parameter estimator $\hat{\alpha}_i, i = 1, 2, 3, ..., n$ for the purpose of synchronizing the master and slave systems in spite of a different chaotic systems which is the extreme case of master/ slave mismatch. To estimate the control functions, we subtract (1) from (2), We define the synchronization error system as the differences between the slave system(2) and the controlled master system. Let us define the error variables between the slave system (2) that is to be controlled and the controlling master system(1) as

$$e = y - x$$

then the error dynamics is obtained as

$$\dot{e} = Ae + (f(y) - f(x)) + u \tag{3}$$

where u is the controller to the system with parameter estimator $\hat{\alpha}_i$. The parameter estimation error is defined as

$$e_{\alpha_i} = \alpha_i - \hat{\alpha}_i, i = 1, 2, 3, ..., n.$$

The synchronization error system controls a controlled chaotic system with control input $u_i, i = 1, 2, 3, ..., n$with adaptive update law $\dot{\hat{\alpha}}_i$ as a function of the parameter estimator error states $e_{\alpha_1}, e_{\alpha_2}, e_{\alpha_3}, ..., e_{\alpha_n}$. That means the systematic adaptive feedbacks so as to stabilize the error dynamics (3), $e_1, e_2, e_3, ..., e_n$ converge to zero as time t tends to infinity. This implies that the controllers $u_i, i = 1, 2, 3, ..., n$ and adaptive update law $\dot{\hat{\alpha}}_i$ should be designed so that the two chaotic systems can be synchronized. In mathematically $lim_{t\to\infty}\|e(t)\| = 0$.

Adaptive control design is systamatic and guarantees global stabilities performance of strict-feedback nonlinear systems. By using the adaptive control design, the chaotic system is stabilized with respect to a Lyapunov function V, by the design of parameter estimator control $\hat{\alpha}_i$ and a control input function u_i with adaptive update law $\dot{\hat{\alpha}}_i$.

The Lyapunov stability approach consists in finding an update law. Lyapunov function technique be methodology. Consider candidate Lyapunov function as

$$V(e, e_\alpha) = e^T P_1 e + e_\alpha^T P_2 e_\alpha \tag{4}$$

where P_1 and P_2 are positive definite matrix.

The parameters of the master and slave systems are estimate and the states of both systems (1) and (2) are measurable. If we find a controller u and adaptive update law $\dot{\hat{\alpha}}_i$ such that

$$\dot{V}(e, e_\alpha) = -e^T Q_1 e - e_\alpha^T Q_2 e_\alpha \tag{5}$$

where Q_1 and Q_2 are positive definite matrix, then $V : \mathbb{R}^n \to \mathbb{R}^n$ is a negative definite function.

Thus by a Lyapunov stability theory [32], the error dynamics (3) is globally exponentially stable and satisfied for all initial conditions $e(0) \in \mathbb{R}^n$. Hence, the states of the master and slave systems are globally and exponentially synchronized and the adaptive control law is given by

$$\dot{\hat{\alpha}}_i = G(e) + k_i e_{\alpha_i} \tag{6}$$

where k_i is positive constant, $e = y - x$ is the error vector, and $G : \mathbb{R}^n \to \mathbb{R}^n$ is a continuous vector function with the error as its argument.

3. The System Description

Recently, theoritical design and hardware implementation of different kinds of chaotic oscillators have attracted increasing attention, aiming real world applications of many chaos based technologies andinformation systems. In current research interest in creating various complex multi scroll chaotic attractors by using simplified and generic electrical circuit. Here which we are interested is the n–scroll Chua circuit which is an improved model of chaotic system introduced by Wallace K. S. Tang et al([31],2001) In fact, it is now obvious that can be derived from simplified and generic electrical circuit.

3.1. The n–Scroll Chua system

Chua's system is utilized for the investigation.The dynamical equation of n–scroll Chua system with sine function([31], 2001) is given by

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{7}$$

where $f(x_1)$ is given by $f(x_1) = \begin{cases} \frac{b\pi}{2a}(x_1 - 2ac) & \text{if} \quad x_1 \geq 2ac \\ -b\sin(\frac{\pi x_1}{2a} + d) & \text{if} \quad -2ac \leq x_1 \leq 2ac \\ \frac{b\pi}{2a}(x_1 + 2ac) & \text{if} \quad x_1 \leq -2ac \end{cases}$.

The piecewise linear function is only nonlinearity in the system. A sine function is couched to obtain the nonlinearity needed for generating chaos in Chua system.

When $\alpha = 10.814, \beta = 14.0, a = 1.3, b = 0.11$, 2-scroll, 3-scroll, 4-scroll and 6-scroll attractors are generated with $c = 1, 2, 3,$and 5 respectively, as depicted in Fig. 1(a)–(c). A maximum of six scroll can be observed.

4. Synchronization of identical n–scroll Chua systems via Adaptive Control Design Based on Feedback Control

In this section we apply the adaptive method with novel feedback function for the synchronization of identical Chua system. The equation for Chua's system are

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - f(x_1)) \\ \dot{x}_2 &= x_1 - x_2 + x_3 \\ \dot{x}_3 &= -\beta x_2 \end{aligned} \tag{8}$$

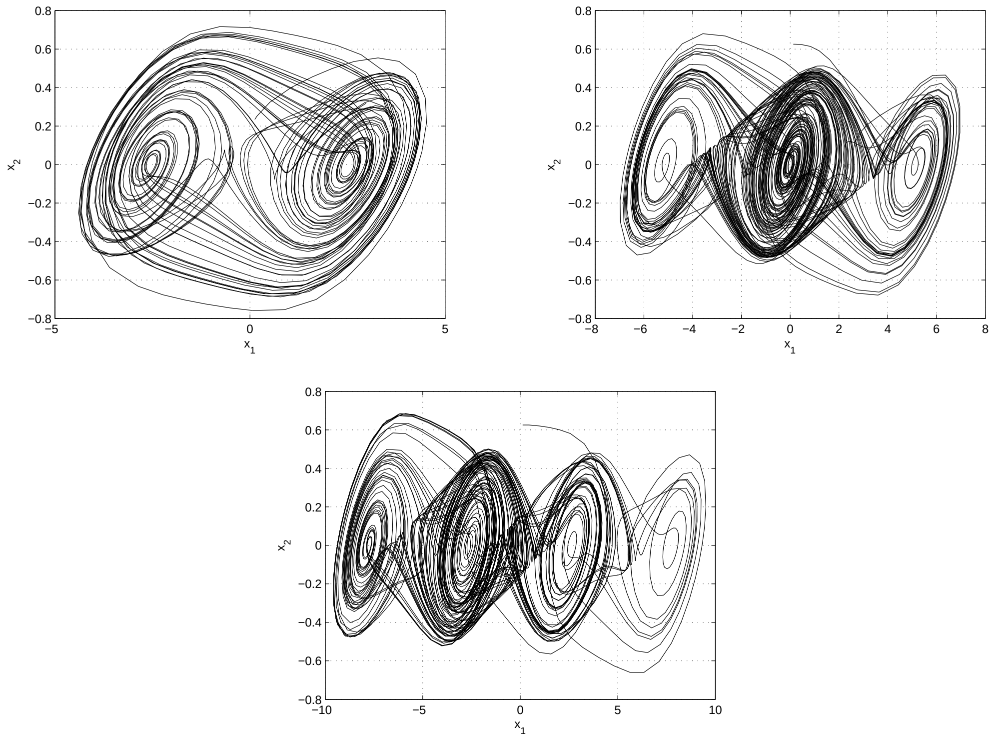

Figure 1: (a). Phase orbit of 2–scroll Chua system when $c = 1$, (b). Phase orbit of 3–scroll Chua system when $c = 2$, (c). Phase orbit of 4–scroll Chua system when $c = 3$,

where $f(x_1)$ is given by $f(x_1) = \begin{cases} \frac{b\pi}{2a}(x_1 - 2ac) & \text{if} \quad x_1 \geq 2ac \\ -b\sin(\frac{\pi x_1}{2a} + d) & \text{if} \quad -2ac \leq x_1 \leq 2ac \\ \frac{b\pi}{2a}(x_1 + 2ac) & \text{if} \quad x_1 \leq -2ac \end{cases}$

where $x(t)(i = 1, 2, 3) \in \mathbb{R}^3$ is a state vectors of the system and the master system also described by Chua system, the system dynamics is

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - f(y_1)) + u_1 \\ \dot{y}_2 &= y_1 - y_2 + y_3 + u_2 \\ \dot{y}_3 &= -\beta y_2 + u_3 \end{aligned} \qquad (9)$$

where $f(y_1)$ is given by $f(y_1) = \begin{cases} \frac{b\pi}{2a}(y_1 - 2ac) & \text{if} \quad y_1 \geq 2ac \\ -b\sin(\frac{\pi y_1}{2a} + d) & \text{if} \quad -2ac \leq y_1 \leq 2ac \\ \frac{b\pi}{2a}(y_1 + 2ac) & \text{if} \quad y_1 \leq -2ac \end{cases}$

where $y(t)(i = 1, 2, 3) \in \mathbb{R}^3$ is a state vectors of the system. Let us define the error variables between the slave system (9) that is to be controlled and the controlling master system(8) as

$$e_i = y_i - x_i, i = 1, 2, 3$$

Subtract (8) from (7) and using the notation (3)yields

$$\begin{aligned} \dot{e}_1 &= \alpha e_2 - \alpha[f(y_1) - f(x_1)] + u_1 \\ \dot{e}_2 &= e_1 - e_2 + e_3 + u_2 \\ \dot{e}_3 &= -\beta e_2 + u_3 \end{aligned} \qquad (10)$$

we introduce the adaptive control to design the controller $u_i, i = 1, 2, 3$. Where $u_i, i = 1, 2, 3$ are control feedbacks, as long as these feedbacks stabilize system (19) converge to zero as the time t goes to infinity.

Let us define the adaptive function u_1, u_2, u_3 as

$$\begin{aligned} u_1 &= -\hat{\alpha}e_2 - \alpha[f(y_1) - f(x_1)] - k_1e_1 \\ u_2 &= -e_1 + e_2 - e_3 - k_2e_2 \\ u_3 &= \hat{\beta}e_2 - k_3e_3 \end{aligned} \tag{11}$$

where $\hat{\alpha}$ and $\hat{\beta}$ are estimates of α and β respectively and $k_i, (i = 1, 2, 3, 4, 5)$ are positive constants.

Substituting eqn(11) into eqn(10), then the error dynamics simplify to

$$\begin{aligned} \dot{e}_1 &= (\alpha - \hat{\alpha})e_2 - \alpha[f(y_1) - f(x_1)] - k_1e_1 \\ \dot{e}_2 &= -k_2e_2 \\ \dot{e}_3 &= -(\beta - \hat{\beta})e_2 - k_3e_3 \end{aligned} \tag{12}$$

Let us define the parameter estimation error as

$$\begin{aligned} e_\alpha &= \alpha - \hat{\alpha} \\ e_\beta &= \beta - \hat{\beta} \\ e_a &= a - \hat{a} \\ e_b &= b - \hat{b} \\ e_c &= c - \hat{c} \end{aligned} \tag{13}$$

substituting (13) into (12), the error dynamics is simplified to

$$\begin{aligned} \dot{e}_1 &= e_\alpha e_2 - \alpha[f(y_1) - f(x_1)] - k_1e_1 \\ \dot{e}_2 &= -k_2e_2 \\ \dot{e}_3 &= -e_\beta e_2 - k_3e_3 \end{aligned} \tag{14}$$

4.1. *case 1: when* $[f(y) - f(x)] \geq 2ac$:

The Lyapunov stability approach consists in finding an update law for tuning the estimates of the parameters, the Lyapunov candidate function is

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_a^2 + e_b^2 + e_c^2) \tag{15}$$

Differentiating equation(15) along the trajectories (12)and using

$$\dot{e_\alpha} = -\dot{\hat{\alpha}}, \dot{e_\beta} = -\dot{\hat{\beta}}, \dot{e_a} = -\dot{\hat{a}}, \dot{e_b} = -\dot{\hat{b}}, \dot{e_c} = -\dot{\hat{c}}$$

we find that

$$\begin{aligned} \dot{V} &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_\alpha(e_1e_2 - \dot{\hat{\alpha}}) + e_\beta(-e_2e_3 - \dot{\hat{\beta}}) \\ &\quad + e_a(-\dot{\hat{a}}) + e_b(\frac{-\alpha\pi}{2a}e_1^2 - \dot{\hat{b}}) + e_c(2\alpha b\pi e_1 - \dot{\hat{c}}) \end{aligned} \tag{16}$$

In equ(16), the parameters are updated by the update law

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1e_2 + k_4e_\alpha \\ \dot{\hat{\beta}} &= -e_2e_3 + k_5e_\beta \\ \dot{\hat{a}} &= k_6e_a \\ \dot{\hat{b}} &= \frac{-\alpha\pi}{2a}e_1^2 + k_7e_b \\ \dot{\hat{c}} &= k_8e_c \end{aligned} \tag{17}$$

substituting eqn(17) into eqn(16), then we have

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_\alpha^2 - k_5e_\beta^2 - k_6e_a^2 - k_7e_b^2 - k_8e_c^2 \tag{18}$$

which is a negative definite function. Thus by a Lyapunov stability theory [32], the error dynamics (3) is globally exponentially stable and satisfied for all initial conditions $e(0) \in \mathbb{R}^8$. Hence, the states of the master and slave systems are globally and exponentially synchronized. Hence, we obtain the following result.

Theorem 1. *The identicaln–scroll Chua's systems (8) and (9) are globally and exponentially synchronized by using adaptive parameter update law (17)with the feedback controls (11) and* $k_i, i = 1, 2, 3, ..., 8$ *are positive constants.*

4.2. case 2: when $-2ac \le [f(y) - f(x)] \le 2ac$*:*

The Lyapunov stability approach consists in finding an update law for tuning the estimates of the parameters, the Lyapunov candidate function is

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_a^2 + e_b^2 + e_c^2) \tag{19}$$

Differentiating equation(15) along the trajectories (12)and using

$$\dot{e_\alpha} = -\dot{\hat{\alpha}}, \dot{e_\beta} = -\dot{\hat{\beta}}, \dot{e_a} = -\dot{\hat{a}}, \dot{e_b} = -\dot{\hat{b}}, \dot{e_c} = -\dot{\hat{c}}$$

we find that

$$\begin{aligned} \dot{V} \quad = \quad & -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_\alpha(e_1e_2 - \dot{\hat{\alpha}}) + e_\beta(-e_2e_3 - \dot{\hat{\beta}}) \\ & + e_a(-\dot{\hat{a}}) + e_b(\alpha e_1[sin(\frac{\pi y_1}{2a} + d) - sin(\frac{\pi x_1}{2a} + d)] - \dot{\hat{b}}) + e_c(-\dot{\hat{c}}) \end{aligned} \tag{20}$$

In equ(16), the parameters are updated by the update law

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1e_2 + k_4e_\alpha \\ \dot{\hat{\beta}} &= -e_2e_3 + k_5e_\beta \\ \dot{\hat{a}} &= k_6e_a \\ \dot{\hat{b}} &= \alpha e_1[sin(\frac{\pi y_1}{2a} + d) - sin(\frac{\pi x_1}{2a} + d)] + k_7e_b \\ \dot{\hat{c}} &= k_8e_c \end{aligned} \tag{21}$$

substituting eqn(17) into eqn(16), then we have

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_\alpha^2 - k_5e_\beta^2 - k_6e_a^2 - k_7e_b^2 - k_8e_c^2 \tag{22}$$

which is a negative definite function. Thus by a Lyapunov stability theory [32], the error dynamics (3) is globally exponentially stable and satisfied for all initial conditions $e(0) \in \mathbb{R}^8$. Hence, the states of the master and slave systems are globally and exponentially synchronized. Hence, we obtain the following result.

Theorem 2. *The identicaln–scroll Chua's systems (17) and (18) are globally and exponentially synchronized by using adaptive parameter update law (21)with the feedback controls (11) and* $k_i, i = 1, 2, 3, ..., 8$ *are positive constants.*

4.3. case 2: when $[f(y) - f(x)] \le -2ac$*:*

The Lyapunov stability approach consists in finding an update law for tuning the estimates of the parameters, the Lyapunov candidate function is

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_\alpha^2 + e_\beta^2 + e_a^2 + e_b^2 + e_c^2) \tag{23}$$

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Differentiating equation(15) along the trajectories (12)and using

$$\dot{e_\alpha} = -\dot{\hat{\alpha}}, \dot{e_\beta} = -\dot{\hat{\beta}}, \dot{e_a} = -\dot{\hat{a}}, \dot{e_b} = -\dot{\hat{b}}, \dot{e_c} = -\dot{\hat{c}}$$

we find that

$$\begin{aligned} \dot{V} &= -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 + e_\alpha(e_1e_2 - \dot{\hat{\alpha}}) + e_\beta(e_2e_3 - \dot{\hat{\beta}}) \\ &\quad + e_a(-\dot{\hat{a}}) + e_b(\frac{\alpha\pi}{2a}e_1^2 - \dot{\hat{b}}) + e_c(-\dot{\hat{c}}) \end{aligned} \tag{24}$$

In equ(16), the parameters are updated by the update law

$$\begin{aligned} \dot{\hat{\alpha}} &= e_1e_2 + k_4e_\alpha \\ \dot{\hat{\beta}} &= e_2e_3 + k_5e_\beta \\ \dot{\hat{a}} &= k_6e_a \\ \dot{\hat{b}} &= \frac{\alpha\pi}{2a}e_1^2 + k_7e_b \\ \dot{\hat{c}} &= k_8e_c \end{aligned} \tag{25}$$

substituting eqn(17) into eqn(16), then we have

$$\dot{V} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_\alpha^2 - k_5e_\beta^2 - k_6e_a^2 - k_7e_b^2 - k_8e_c^2 \tag{26}$$

which is a negative definite function. Thus by a Lyapunov stability theory [32], the error dynamics (3) is globally exponentially stable and satisfied for all initial conditions $e(0) \in \mathbb{R}^8$. Hence, the states of the master and slave systems are globally and exponentially synchronized. Hence, we obtain the following result.

Theorem 3. *The identicaln–scroll Chua's systems (8) and (9) are globally and exponentially synchronized by using adaptive parameter update law (27)with the feedback controls (11) and* $k_i, i = 1, 2, 3, ..., 8$ *are positive constants.*

5. Numerical Simulation

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the differential equations(8) and (9)with the feedback controls u_1, u_2 and u_3 given by(18). The parameters of the systems (17) and (18) are taken in the case of chaotic case as

$$\alpha = 10.814, \beta = 14.0, a = 1.3, b = 0.11, c = 3, d = 0.$$

The initial value of the drive system(17) are chosen as

$$x_1(0) = .125, x_2(0) = .625, x_3(0) = .941$$

and response system(18)are chosen as

$$y_1(0) = 0.321, y_2(0) = 0.487, y_3(0) = 0.965$$

The initial values of the parameter estimates are taken as:

$$\hat{\alpha}(0) = 2, \hat{\beta}(0) = 0.3, \hat{a}(0) = 6, \hat{b} = 8, \hat{c} = 10$$

We take the parameters $k_i = 2, i = 1, 2, 3...., 8$ Fig. 2(a), (b)and (c) depicts the synchronization of identical n–scroll Chua's circuit (8) and (9).

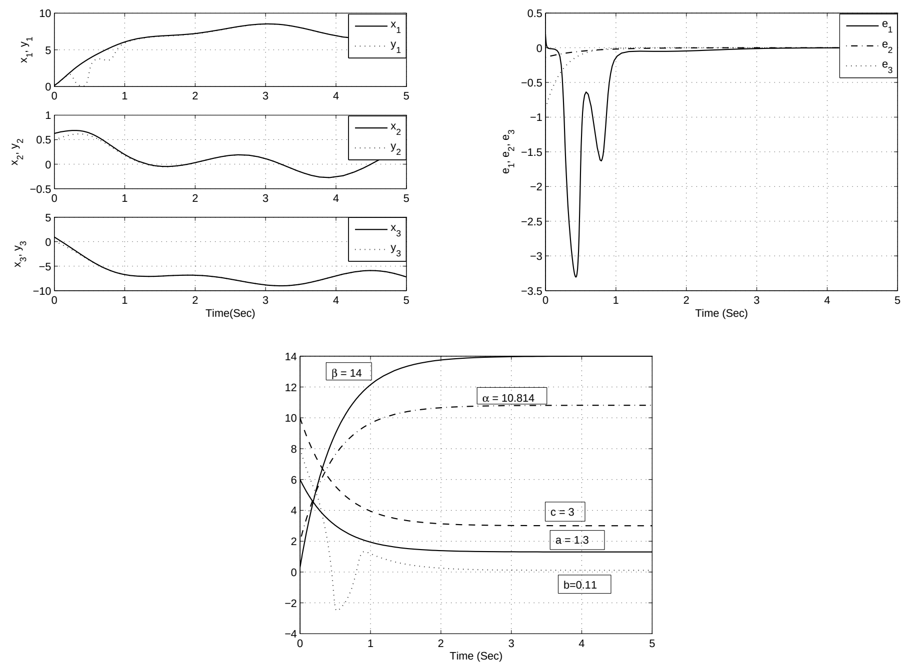

Figure 2: (a). Synchronization of n-scroll Chua circuit (b). Error portrait of n-scroll Chua circuit (c). Parameter estimates of $\hat{\alpha}, \hat{\beta}, \hat{a}, \hat{b}, \hat{c}$

6. Conclusion

In this paper, adaptive control method has been applied to estimate the fixed but unknown parameter and achieve global chaos synchronization for a family of n-scroll chaotic Chua circuit. The advantage of this method is a recursive procedure for synchronizing chaotic system and there is no derivative in controller. The adaptive control design has been demonstrated to family of n-scroll chaotic Chua circuit. Numerical simulations have been given to illustrate and validate the effectiveness of the proposed synchronization schemes of the chaotic circuit. The adaptive control design is very effective and convenient to achieve global chaos synchronization.

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