An Overview of the AHP Algorithm for Multi-Criteria Decision Making

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Abstract--- *This paper deals with decision-making using the Analytic Hierarchy Process (AHP), a Multi-Criteria Decision Making (MCDM) method, developed by Thomas L. Saaty. The hierarchical decomposition approach to decision problems followed by pairwise comparisons of both tangible and intangible attributes using the Saaty scale is described. The eigenvector method and the geometrical mean method to derive priorities are discussed. A numerical example using the AHP methodology is given and the dilemma that ranks already derived can change when a new alternative is added is shown. Although the AHP procedure is extremely widely used, it has been subject to criticism on several counts. Problems with the numerical scale used in AHP and the difficulty of quantifying intangible factors are highlighted. This paper introduces the AHP methodology and draws attention to controversies and open issues surrounding the algorithm.*

Keywords--- *Multi-Criteria Decision Making, Analytic Hierarchy Process, Pairwise Comparison, Eigenvector, Rank Reversal, Ratio Scale, Zero, Unit*

I. INTRODUCTION

ANALYTIC HIERARCHY PROCESS (AHP) is a Multi-Criteria Decision Making (MCDM) technique, which was developed by Thomas L. Saaty from 1972 onwards. This method is partly a subjective approach with an attempt to incorporate logic, psychology, and past experiences. It tries to reduce a complex problem into a more structured format that is hopefully easier to comprehend. This method comprises a hierarchy of levels which consists of Goal, Criteria, Sub-Criteria, and Alternatives. This technique consists of a set of axioms which helps define a set of matrices and assigns priority through pairwise comparisons to criteria and alternatives. It is applied in many engineering, scientific and commercial fields. In the field of medicine it has been used for drug selection, choosing the right organ transplant recipients, fertility treatment options, for choosing between angioplasty and coronary bypass surgery, etc[12][15][24][25][32]. AHP has been used in applications such as nuclear waste management and other environmental studies [19, 26]. It has been used to rank sports teams [11]. It has been applied to computer science areas such as selection of operating systems, embedded systems, selection of COTS software, etc [1][7][13]. A general survey on AHP applications has been carried out, among others, by Omkarprasad S. Vaidya, Sushil Kumar (2004) [31]. One of the most famous uses of the Eigenvector approach that AHP relies on is the PageRank algorithm used by Google for ranking websites [14]. A pre-requisite is a decision-maker who can understand the domain of the problem. Thus AHP can be considered to be one of the Multi-Criteria Decision Making methods for solving certain complex problems. Other approaches to such problems include outranking methods, multi-attribute utility theory, etc. However, the AHP method is the most widely used and researched approach with over 25,000 citations in the literature. Despite its widespread use many aspects of AHP remain controversial in nature. In particular the problem of reversal of ranks when an alternative is added or deleted has attracted attention [3, 16].

II. GENESIS OF AHP

In 1960, T. Saaty, a mathematician who has made several contributions in operations research, arms control and disarmament, and urban design was asked to lead a special research project for the Arms control and Disarmament Agency at the U.S Department of State. A large budget enabled him to hire experts in different fields including lawyers. The insight he gained from the poor results led him to conclude that even experts did not have a good practical approach to problems and were faced with communication difficulties and different perspectives. He was also convinced of the limitations of conventional decision making approaches such as utility theory. This observation made him seriously think about the modalities of decision making and prompted him to develop a method that would help even lay people take complicated decisions. That led him to the basics of the Analytic Hierarchy Process (AHP)[6].

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2.1 The Basic Practices when using AHP

(a)Problem representation

(b)Pairwise Comparison

(c)Developing Local Priority

(d)Checking Consistency

(e)Determining Global Priority

(f) Sensitivity Analysis.

(a) Problem Representation

With all decision making processes the decision maker is assumed to be a knowledgeable person who structures a problem that can be divided into sections: objective, criteria, sub-criteria (if required) and alternatives. AHP leads to a hierarchical structure of objectives, criteria, sub-criteria and alternatives which help a decision-maker reduce the complexity of a problem. It focuses on the criteria, and the alternatives with respect to criteria, for assigning the priorities in a near-consistent matrix using a pairwise comparison approach.

(b) Pairwise Comparison

At the core of the AHP approach are pairwise comparisons of both tangible and intangible factors to arrive at decisions. In AHP the data are derived purely by pairwise comparisons between alternatives with respect to an independent criterion. Saaty states that the verbal declarations are converted to numerical numbers for quantification in accordance with the fundamental scale given by him (and keeping Axiom 2 – discussed below – in mind) as given in Table 1 below.

Although a number of other scales have been proposed, the Saaty scale is most frequently used. Some other controversial aspects of the Saaty scale are taken up in Section XI, Problematic Aspects of AHP.

III (c), (d), and (e) are discussed further below in Section V, AHP Algorithm.

Table 1: [Fundamental Scale of Relative Importance According to Saaty (1980)]

Scale for pairwise Comparisons		
Intensity of Importance	Definition	Explanation
1	Of equal value	Two requirements are of equal value
3	Slightly more Value	Experience slightly favours one requirement over another
5	Essential or Strong value	A requirement is strongly favoured and its dominance is demonstrated in practice
7	Very Strong Value	A requirement is very strongly favoured and its dominance is demonstrated in practice
9	Extreme value	The evidence favoring one over another is of the highest possible order .
2,4,6,8	Intermediate values between two adjacent judgements	When a compromise is needed
Reciprocals	If requirement i has one of the above numbers assigned to it when compared with requirement j, then j has the reciprocal value when compared with i.	

2.2 Axioms of AHP

These axioms were defined by T.Saaty to improve the quality of the final outcome and derived from criticisms from different researchers [28][29].

Axiom 1 is named as the Reciprocal axiom; it states that if during a pairwise comparison, A is strongly preferred when compared to B , i.e., if A=nB, then B is (1/n) times as preferred as A. B = (1/n)A. (i.e., $a_{ij} = 1/a_{ji}$, where i defines the row, and j defines the column of the matrix.)

Axiom 2, named as the homogeneity axiom, states that the elements being compared should differ by only a few factors; otherwise, there will be definitely an error in the judgement. i.e., if both these two elements are entirely different, say one element is a river and the other is an ocean, then there is no meaning in the comparison. If both these elements are comparatively different, i.e., say one element being an absolutely excellent grade in a subject and the other element a very poor grade, then there is no point in performing the comparison.

Axiom 3, named as synthesis axiom, states that the elements lower down the hierarchy do not influence elements at higher levels, i.e. such reverse feedback is not allowed.

Axiom 4, named as the expectation axiom, states that it is necessary to ensure that the decision-maker's thoughts are properly reflected and influence how the problem is structured and pairwise comparisons are carried out in order that the final outcome conforms to the innate beliefs of the decision maker.

2.3 AHP Algorithm

Step (1): Represent the problem as a decision hierarchy containing the Goal, Criteria and Alternatives.
Step (2): Construct a pairwise comparison n x n matrix for the n criteria.
Step (3): Determine the dominance of each criterion by making a series of judgements using pairwise comparisons of criteria. This is the judgement matrix.
Step (4): Determine the priority vector. There are several methods to obtain the priority vector. Among them:
a) Saaty's Eigenvector method.
Raise the pairwise comparison matrix to a sufficiently large power.
Sum over rows and normalize to get an estimate of the eigenvector.
Stop when there is a very small difference between the components of two successive estimates of the eigenvector.
b) The geometric mean method (i.e. the logarithmic least squares method):
Take the geometric mean of each of the rows of the pairwise comparison matrix and normalize.

Determine the maximum Eigenvalue (λ_{max}) of the pairwise comparison matrix: Calculate the product of the vector of the total of each column of the judgement matrix with the corresponding priority vector .
Step (5): Calculate the Consistency Index (C.I) using the formula C.I = (λ_{max} - n) /(n-1)
Step (6): Calculate the Consistency Ratio (C.R) using the formula C.R = C.I/R.I. The Random Index (R.I) value can be taken from the following Table 1:

Table 1: (RI values for different values of n, where n is the order of the matrix)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0.0	0.0	0.58	0.9	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

Step (7): Check the consistency of the n x n matrix; if the C.R > 0.1, then reevaluate the pairwise comparison , perform Step (2)
Step (8): Construct a set of pairwise comparison matrices for the alternatives with respect to the criterion; consider n criteria and m alternatives, then n number of matrices are constructed of order m x m.
Step (9): Determine the dominance of each alternative by making a series of judgements using pairwise comparisons of two alternatives at a time.
Step (10): Perform Step (4) to determine λ_{max} , Step (5) to determine C.I , Step (6) to determine C.R.
Step (11): Check the consistency of the m x m matrix (alternatives with respect to each criterion); if the C.R > 0.1, then re-evaluate the pairwise comparison, perform Step (9)
Step (12): The Final Priority (Global Priority) for each alternative is determined by summing the product of the criteria weights and the corresponding local priority of an alternative with respect to each criterion, by using the formula

$$\text{Global Priority} = \sum_{j=1}^{n} a_{ij} * w_j \quad , \text{for } i= 1,2,3\text{……….}m ,$$

where a_{ij} is the $(i,j)^{th}$ element of the decision matrix formed with the columns of the priority vectors and w_j is the weight of the corresponding criterion.

2.4 Some Mathematical Basics in AHP:

- *Eigenvector Method*

While there are many methods for obtaining the priority vector, Saaty has claimed that the Eigenvector method is superior to all other methods in preserving rank order.

- *Consistency of Matrix*

In the pairwise comparisons, if the reciprocity property holds and if A is preferred to B and B is preferred to C means A is preferred to C then the matrix is consistent. A matrix is consistent if and only if the principal eigenvalue λ_{max} = n , the order of the matrix . The sum of the eigenvalues of a matrix is equal to its trace, the sum of its diagonal elements, and in this case the trace of A is equal to n. Thus all its eigenvalues except one are zero. In actual practise we allow for a small amount of inconsistency (Consistency Ratio (C.R) less than 10 percent) and λ_{max} will be slightly greater than the value n.

Justification of Saaty's Eigenvector Approach

Now , a property of a consistent matrix A is that the condition $A^k = n^{k-1} A$, where n is the order of A, holds .But this condition does not apply for dominance of an inconsistent matrix .Instead , as Saaty points out , one must consider priorities got from direct dominance of object i over object j from the (i,j)th element in the matrix A, second order dominance of object i over object j from the (i,j)th element in A^2 which gives the sum of the dominance intensities of all 2-walks from object i to object j , etc. [21].

The total dominance of each object is got by normalizing the sum of its rows. Thus the i^{th} value of the Perron priority vector got from A^k can be viewed as a dominance of object i over other objects along all k-walks i.e. all k length paths from object i to other objects [21]. This results in a series of priority vectors each giving a different level of dominance.

The limit of the average of this series of priority vectors is the same as the limit of the sequence of powers of the matrix.

Finally ,from a result due to Perron this sequence A^k converges to a matrix whose columns after normalization are identical and yields the principal eigenvector of A to within a factor of proportion [21 ,22].

Hence , it is claimed , follows the validity of Saaty's method.

Geometric Mean Method (i.e. the logarithmic least squares method):

Critics have, however, pointed out that the ranking by the eigenvector approach is mainly ordinal in the AHP method as the actual values of pairwise coefficients play a lesser role than their relative values [18]. Many researchers have hence argued for the geometric mean method (i.e. the logarithmic least squares method) to obtain the priority vector and it has been claimed that this method does not have the drawback of rank reversal [17]. Saaty has claimed the opposite and argues that the geometric mean method can lead to rank reversal [23].

The two methods give the same priority vector for 3 by 3 matrices. An advantage of the geometric mean method is that the geometric means of the rows and the columns give the same ranking. The eigenvector approach is known to have the problem of rank reversal if the left rather than the right eigenvector were to be used [10]. Furthermore, some feel the eigenvector approach lacks clarity [10]. This lack of clarity also leads to an opaque and mechanical process of churning out priority values, especially by novices in this area.

2.5 A Numerical Example

A Decision Maker (DM) is faced with the problem of purchasing one computer out of a choice of a total of three computers, hereafter called Comp 1, Comp 2 and Comp 3. The Criteria selected as pertinent by this person are Cost, Warranty, and Support. According to Saaty [20], the scale expresses an order, based on the decision maker's attitude. It is important to stress that for Saaty intangible factors like the level of support or comprehensiveness of warranty provided are also extremely important. Further, and controversially as argued in the Section XI of this paper, pairwise comparisons are claimed to measure the intensity or degree by which one computer is better than another on these intangible factors .

Consider Table 3:

The decision maker compares the criterion 'Cost' with another criterion 'Warranty', as well as s/he compares 'Cost' with 'Support'. From the matrix given in Table 3, for this particular decision maker the Cost criterion ' is extremely important ' when compared to Warranty. From this matrix, the criterion Cost = 9 x (Warranty) and Warranty = (1/9) x Cost. The matrix given in Table 3 is perfectly consistent, if and only if the transitivity rule (1) and reciprocity rule (2) hold good for all the comparisons of a_{ij}

$$a_{ij} = a_{ik} * a_{kj} \tag{1}$$

$$a_{ij} = 1/a_{ji} \tag{2}$$

The following are the judgement matrices along the calculated priority vectors with three alternatives in terms of a criterion; here the geometric mean method is used to obtain the priority vectors:

For instance:

The judgement matrix with respect to the 'Cost' criterion is Table 4;

The judgement matrix with respect to the 'Warranty' criterion is Table 5;

The judgement matrix with respect to the 'Support' criterion is Table 6.

After the alternatives are compared with each other in terms of each one of the decision criteria the individual priority vectors are derived. The priority vectors become the columns of the decision matrix (shown in Table 7). The weights of importance of the criteria are also determined by using pairwise comparisons. Therefore, if a problem has M alternatives(here M=3) and N criteria (here N=3), then the decision maker is required to construct N judgment matrices (one for each criterion) of order MxM and one judgment matrix of order NxN (for the N criteria). The judgement matrix of order N x N (for the N criteria, here N=3) is given in Table 3 which shows the judgement matrix when comparing the significance of the three decision criteria. Global Priority derived by aggregating local priorities is shown in Table 7. Thus the ranks are Comp 2 > Comp 3 > Comp 1

2.6 Phenomenon of Rank Reversal

As already mentioned AHP suffers from the problem of rank reversal. The paper by Belton and Gear showed that ranks can change if even a copy of an existing alternative were to be added to the choice set [3]. Subsequent studies revealed that ranks can alter if a near alternative were to be added. A lively debate has since ensued with, however, no consensus on basic issues such as the causes of rank reversal or even whether rank reversal is a natural, not wholly undesirable phenomenon or something to be avoided at all cost [11,21]. While Saaty and his group have sought to justify the reversal of ranks in particular cases and have even claimed as an advantage that AHP used appropriately allows for rank reversals , other researchers find rank reversals a major problem .

If a new alternative Computer 4 is added, then the following are the judgement matrices with four alternatives in terms of a criterion :

For instance:

The judgement matrix with respect to the 'Cost' criterion is Table 8 ;

The judgement matrix with respect to the 'Warranty' criterion is Table 9;

The judgement matrix with respect to the 'Support' criterion is Table 10.

The Global Priority derived by aggregating local priorities is given in Table 11 .Thus it is seen from the above calculation that the revised ranks are Comp 3 > Comp 2 > Comp 1 > Comp 4. It is thus seen that the ranks have changed.

Saaty would claim that this reversal is a consequence of the use of the geometric mean method of calculating the priority vectors. However, much of the force of this argument is taken away by the problems with Saaty's AHP that are discussed in Section XI below.

Belton - Gear (1983) [3] proposed a revised version of the original AHP called revised-AHP (RAHP) because of the problem of rank reversal in the original AHP . In this method each column of the global priority decision matrix is to be divided by the maximum value of that column. Afterward, Saaty (1994) accepted this variant of AHP and it is called the Ideal Mode AHP [29]. The Global Priority derived from local priorities by aggregation is given in Table 12. The ranks are Comp 2 > Comp 3 > Comp 1.

If a new alternative Comp 4 is added, then with four alternatives using the Belton – Gear Method (Revised AHP or RAHP) , the Global Priority derived from local priorities by aggregation is given in Table 13 .

III. RESULT:

For three alternatives the final result is shown in Table 14 .If the new alternative Comp 4 is added to the existing three alternatives, then the final result is shown in Table 15. Thus it is seen from the calculation that the revised ranks are Comp 3 > Comp 2 > Comp 1 > Comp 4. The results from Table 14 and Table 15 indicate that rank reversal occurs in both the original AHP and the revised-AHP (RAHP) when a new alternative Comp 4 is added.

3.1 List of Tables

Table 3: Judgement Matrix for Criteria

	Cost	Warranty	Support	Priority Vector(PV)
Cost	1.000	9.000	5.000	0.769
Warranty	0.111	1.000	1.000	0.104
Support	0.200	1.000	1.000	0.127
Sum	1.311	11.000	7.000	1.000
Sum x PV	1.008	1.144	0.889	
(λ max = 3.041, CI = 0.021, CR= 0.036)				

Table 4: The Judgement Matrix with Respect to the 'Cost' Criterion

Cost	Comp 1	Comp 2	Comp 3	Priority Vector(PV)
Comp 1	1.000	0.125	0.200	0.073
Comp 2	8.000	1.000	1.000	0.500
Comp 3	5.000	1.000	1.000	0.427
Sum	14.000	2.125	2.200	1.000
Sum x PV	1.022	1.063	0.939	
(λ max = 3.024, CI = 0.012, CR= 0.021)				

Table 5: The Judgement Matrix with Respect to the 'Warranty' Criterion

Warranty	Comp 1	Comp 2	Comp 3	Priority Vector(PV)
Comp 1	1.000	5.000	5.000	0.714
Comp 2	0.200	1.000	1.000	0.143
Comp 3	0.200	1.000	1.000	0.143
Sum	1.400	7.000	7.000	1.000
Sum x PV	1.000	1.001	1.001	
(λ max = 3.002, CI = 0.001, CR= 0.002)				

Table 6: The Judgement Matrix with Respect to the 'Support' Criterion

Support	Comp 1	Comp 2	Comp 3	Priority Vector(PV)
Comp 1	1.000	4.000	4.000	0.667
Comp 2	0.250	1.000	1.000	0.167
Comp 3	0.250	1.000	1.000	0.167
Sum	1.500	6.000	6.000	1.001
Sum x PV	1.001	1.002	1.002	
(λ max = 3.005, CI = 0.003, CR= 0.004)				

Table 7: Global Priority derived by aggregating local priorities

Criteria	Cost	Warranty	Support	Score
Options	0.769	0.104	0.127	
Comp 1	0.073	0.714	0.667	0.215
Comp 2	0.500	0.143	0.167	0.421
Comp 3	0.427	0.143	0.167	0.364

Table 8: The Judgement Matrix with Respect to the 'Cost' Criterion

Cost	Comp 1	Comp 2	Comp 3	Comp 4	Priority Vector(PV)
Comp 1	1.000	0.125	0.200	1.000	0.074
Comp 2	8.000	1.000	1.000	3.000	0.413
Comp 3	5.000	1.000	1.000	5.000	0.418
Comp 4	1.000	0.333	0.200	1.000	0.095
Sum	15.000	2.458	2.400	10.000	1.000
Sum x PV	1.110	1.015	1.003	0.950	
(λ max = 4.078, CI = 0.026, CR= 0.029)					

Table 9: The Judgement Matrix with respect to the 'Warranty' Criterion

Warranty	Comp 1	Comp 2	Comp 3	Comp 4	Priority Vector(PV)
Comp 1	1.000	5.000	5.000	1.000	0.449
Comp 2	0.200	1.000	1.000	0.333	0.102
Comp 3	0.200	1.000	1.000	0.333	0.102
Comp 4	1.000	3.000	3.000	1.000	0.348
Sum	2.400	10.000	10.000	2.666	1.001
Sum x PV	1.078	1.020	1.020	0.928	
(λ max = 4.046, CI = 0.015, CR= 0.017)					

Table 10: The Judement Matrix with respect to the 'Support' Criterion

Support	Comp 1	Comp 2	Comp 3	Comp 4	Priority Vector(PV)
Comp 1	1.000	4.000	4.000	1.000	0.433
Comp 2	0.250	1.000	1.000	0.250	0.108
Comp 3	0.250	1.000	1.000	0.333	0.153
Comp 4	1.000	4.000	1.000	1.000	0.306
Sum	2.500	10.000	7.000	3.250	1.000
Sum x PV	1.083	1.080	1.071	0.995	
(λ max = 4.229, CI = 0.076, CR= 0.085)					

Table 11: Global Priority

Criteria	Cost	Warranty	Support	Score
Options	0.769	0.104	0.127	1.000
Comp 1	0.074	0.449	0.433	0.159
Comp 2	0.413	0.102	0.108	0.342
Comp 3	0.418	0.102	0.153	0.351
Comp 4	0.095	0.348	0.306	0.148

Table 12: Global Priority (using Belton-Gear method)

Criteria	Cost	Warranty	Support	Final Priority	After Normalization
Options	0.769	0.104	0.127		
Comp 1	0.146	1.000	1.000	0.343	0.183
Comp 2	1.000	0.200	0.250	0.822	0.439
Comp 3	0.854	0.200	0.250	0.709	0.378

Table 13: Global Priority (using Belton-Gear method)

Criteria	Cost	Warranty	Support	Final Priority	After Normalization
Options	0.769	0.104	0.127		
Comp 1	0.177	1.000	1.000	0.367	0.162
Comp 2	0.988	0.227	0.249	0.815	0.360
Comp 3	1.000	0.227	0.353	0.837	0.370
Comp 4	0.095	0.775	0.707	0.243	0.107

Table 14: Final Result for Three

Rank	Original AHP	Ideal Mode AHP (RAHP)
1	Comp 2	Comp 2
2	Comp 3	Comp 3
3	Comp 1	Comp 1

Table 15: Final Result for Four Alternatives Alternatives

Rank	Original AHP	Ideal Mode AHP (RAHP)
1	Comp 3	Comp 3
2	Comp 2	Comp 2
3	Comp 1	Comp 1
4	Comp 4	Comp 4

3.2 *Problematic Aspects of AHP*

Thus it is seen that AHP is a very widely used decision-making method despite drawbacks such as the problem of rank reversal and questions about the appropriateness of the numerical scale used in AHP [9,19,33]. Triantaphyllou and Mann caution that "when the AHP or the revised AHP methods are used in combination with the eigenvalue approach for processing the input data, then severe alternative ranking failures are possible " [5]. There has been no consensus in the literature on the causes of rank reversal [16, 34]. Controversy and open issues remain on the question of measurement scales, the choice of method for calculating priority vectors, and the appropriate method to aggregate local priorities into a global one.

Dodd and Donegan drew pointed attention to the weaknesses of AHP:

1. The Saaty scale is vague and not closed under multiplication.
2. Saaty uses arithmetic which is not valid.
3. Saaty fails to appreciate the vital difference between values taken from an interval scale and the ratio scale.
4. Saaty ignores the impact of rounding judgement ratios into numbers [5].

Consider an example where a teacher finds student B to be slightly better than student A and C to be slightly better than C. Although only a small difference separates the two pairs of students, the ratio between students A and C could be 9 on Saaty's scale indicating an extremely better performance by C as compared to A.

Similarly, if two successive pair wise comparisons yield values of 3 and 4 respectively, their product results in the value of 12, a value that is outside the 1 to 9 range of the Saaty scale .

The importance attached by researchers to the issue of rank reversal has, perhaps, served to deflect attention from other fundamental problems with the AHP approach. At the very core of the AHP methodology is the claim that pairwise comparisons of both tangible and intangible factors can be made leading to a relative ratio scale. This ratio scale, by Saaty's own admission, lacks both a zero value and a unit value [23]. Saaty's ratio scale thus is quite different from the common usage of the term " ratio scale " in the literature. Thus in ranking cars for purchase both tangible factors like the cost , mileage obtainable ,and intangible factors like the intensity of preference based on the quality of service provided by the dealer , the aesthetic appeal of the car , etc. could be taken into account . To quote Saaty:"We need a way to quantify feelings and intensities of feelings .The ability to do that *(something thought to be impossible by most people)* allows us to measure a crucial factor in decision making ... [27] (italics added for emphasis) .

It is not really permissible to conclude using a scale that lacks both a true zero value and a unit measure that the intensity of one's preference for A is some multiple of that of B [34]. On the Centigrade scale that is without a true zero, for example , it is not right to conclude that city A at 30^0 Centigrade is twice as hot as city B at 15^0 Centigrade .

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Despite the fact that Saaty himself concedes that the vast majority do not agree with his view on measurement of intangibles, most researchers have side-stepped such fundamental issues and this has led to the huge number of citations to his work.

According to Saaty, the scale used in AHP is a linear one. Human feelings are converted into numerical numbers, i.e., quantified in order to suit this scale. Human feelings differ from person to person. It is difficult to quantify human feelings [8]. To quote the mathematicians Davis, P.J. and R. Hersh, "If you are more of a human being, you will be aware there are such things as emotions, beliefs, attitudes, dreams, intentions, jealousy, envy, yearning, regret, longing, anger, compassion and many others. These things -- the inner world of human life -- can never be mathematized"[4]. (Quoted by Saaty in [23]. Saaty, of course does not agree with this view, and believes that even intangibles and feelings can be quantified).

Saaty can be given credit for spreading awareness that the eigenvector approach can be used for ranking phenomena. His work may have even contributed in a limited way to the development of the PageRank algorithm of Google [14]. Nor can one really question Saaty's effort to bring mental phenomena within the domain of science. He cites physicists like David Bohm and Arthur Eddington and even Swami Muktananda in support of his quest[27]. But his claim through his extensive writings that mental phenomena can be measured deserve a closer critical scrutiny.

Other researchers have introduced modifications to the original AHP. These methods such as the multiplicative AHP (MAHP) are also seen to suffer from problems [2][30].

Despite these drawbacks, the initial phase of AHP may still serve the purpose of helping to understand the problem structure. So long as the numbers or numerical results are interpreted with extreme caution, the hierarchical decomposition procedure may throw light on the problem domain. Thus the hierarchical decomposition phase of AHP used along with other approaches and a holistic view may help clarify a decision situation. In deciding mode of transport for cities, for e.g., if pollution control or sustainability were also included as criteria , the solution alternatives may comprise not only cars but also public transport or even bicycles. As Milan Zeleny points out it is often necessary to move beyond taking a decision by choosing among different recipes of bread and ask "why bread and not croissant?" [35]. However, in many applications of the AHP that have been reported in the literature such a holistic view of criteria and applications appear to be missing.

IV. CONCLUSION

Despite the appearance of a logical and mathematical approach coupled with the claim that even intangibles can be measured, it is necessary to view Saaty's AHP with both caution and skepticism. It is known that other multi-criteria decision making methods also have their limitations. The many rival schools of thought in this area, such as Utility theory, Out Ranking methods and Saaty's AHP differ in their basic premises, and hence lead to different solutions when applied to the same decision situation. Human decision making in highly uncertain environments will remain at least partly an art not wholly amenable to subjugation by quantification.

Perhaps it would be appropriate to end by noting Einstein's observation in this context: "Not everything that counts can be counted and not everything that can be counted counts ".

REFERENCES

[1] Ahmad,N, PA Laplante (2009), Using the analytical hierarchy process in selecting commercial real-time operating systems, International Journal of Information Technology & Decision Making Volume 08, Issue 01, pp.151 -168 , World Scientific Publishing Co.

[2] Barzilai, J, and F.A.Lootsma (1997), 'Power Relations and Group Aggregation in the Multiplicative AHP and SMART', Journal of Multi Criteria Decision Analysis, Vol 6, pp. 155 – 165.

[3] Belton, V. and Gear, T. (1983) 'On a shortcoming of Saaty's method of analytic hierarchies', Omega, 11,pp. 228-230.

[4] Davis, P.J. and R. Hersh, (1986). Descartes Dream, Harcourt Brace and Jovanovich, New York, pp.321.

[5] Donegan, H., Dodd, F., & McMaster, T. (1992). A new approach to AHP decision-making. The Statician 41, pp. 295-302.

[6] Ernest H. Forman, Saul I.Gass. (2001) 'The Analytic Hierarchy Process – An Exposition', Operations Research, (INFORMS), 49(4), pp. 469 – 486.

[7] Garg, P, A. Gupta, and J.W. Rozenblit, (2004). Performance Analysis of Embedded Systems in the Virtual Component Co-Design Environment. Proceedings of the 11th IEEE International Conference and Workshop on the Engineering of Computer-Based Systems (ECBS'04) .2004 IEEE, pp. 61-68.

[8] Hand ,D J(1996) 'Statistics and the Theory of Measurement', Journal of the Royal Statistical Society. Series A(Statistics in Society),Vol. 159, No.3, pp.445 – 492.

[9] Alessio Ishizaka, Ashraf Labib.(2011)'Review of the main developments in the Analytic Hierarchy Process', Expert Systems with applications,38(11), pp. 14336-4345.

[10] Johnson, C., Beine, W., & Wang, T. (1979). Right-Left Asymmetry in an Eigenvector Ranking Procedure. Journal of Mathematical Psychology, 19, pp.61-64.

[11] James P.Keener,(1993). The Perron- Frobenius Theorem and the Ranking of Football Teams, SIAM Review, Vol.35, No.1, pp.80 – 93.

[12] Tom Koch,(1996). Normative and Prescriptive Criteria: The Efficacy Of Organ Transplantation Allocation Protocols. Theoretical Medicine 17:75-93.

[13] Jyrki Kontio1, (1996).A Case Study in Applying a Systematic Method for COTS Selection, 1996 IEEE. Published in the Proceedings of the 18th International Conference on Software Engineering (ICSE-18), March 25-29, Berlin, Germany, pp.201-209.

[14] Amy N. Langville and Carl Dean Meyer, (2012). Who's # 1 ? : The Science of Rating and Ranking .Princeton University Press, p. 51.

[15] Matthew J. Liberatore, Robert L. Nydick,(2008). The analytic hierarchy process in medical and health care decision making: A literature review .European Journal of Operational Research 189, pp.194–207.

[16] Hamed Maleki , Sajjad Zahir ,(2012). A Comprehensive Literature Review of the Rank Reversal Phenomenon in the Analytic Hierarchy Process . J. Multi-Crit. Decis. Anal.,Vol 20, Issue 3-4,pp.141-155.

[17] David Olson .(1996) Decision Aids for Selection Problems , Springer . , p 72

[18] Jean-Charles Pomerol, Sergio Barba Romero (2000). Multicriterion Decision in Management: Principles and Practice. Kluwer Academic Publishers , p 98

[19] Ramanathan.R (2001) 'A note on the use of the Analytic Hierarchy Process for the environmental impact assessment', Journal of Environmental Management. 63, 27-35.

[20] Thomas L.Satty. (2008) 'Decision Making with the Analytic Hierarchy Process', International Journal Services Sciences vol.1, no.1., pp. 83-98 .

[21] Thomas L Saaty (1987). Rank According to Perron: A New Insight. Mathematics Magazine, 60/4, pp. 211-213.

[22] Thomas L Saaty (1994). Fundamentals of Decision Making and Priority Theory with the Analytic Hierarchy Process. Pittsburgh, PA: RWS Publications, pp 527.

[23] Thomas L Saaty (2008) ,Relative Measurement and Its Generalization in Decision Making …RACSAM R. Acad. Cien. Serie A. Mat. VOL. 102 (2), pp. 251–318.

[24] Thomas L Saaty, LG Vargas, (2012). Infertility Decision Making - Chapter 23 in Models, Methods, Concepts & Applications of the Analytic Hierarchy Process, pp. 323-330, Springer.

[25] Thomas L Saaty, LG Vargas,(2012).Deciding between angioplasty and coronary artery bypass surgery - Chapter 24 in Models, Methods, Concepts & Applications of the Analytic Hierarchy Process, pp. 331-341 , Springer.

[26] Thomas L Saaty, H Gholamnezhad, (1982). High-level nuclear waste management: analysis of options, Environment and Planning B, 9(2), pp.181-196.

[27] Thomas L Saaty,(2010). Principia Mathematica Decernendi: Mathematical Principles of Decision Making, Pittsburgh, Pennsylvania, RWS publications, pp. 8-9.

[28] Evangelos Triantaphyllou, Stuart H.Mann. (1994), 'A Computational Evaluation Of The Original And Revised Analytic Hierarchy Process' , Computers and Industrial Engineering, Vol. 26, No. 3, pp. 609-618.

[29] Evangelos Triantaphyllou, Stuart H.Mann. (1995), 'Using the Analytic Hierarchy Process for Decision making in Engineering Applications: Some challenges', International Journal of Industrial Engineering: Applications and Practice, vol.2, no.1, pp.35 – 44.

[30] Evangelos Triantaphyllou.(2001) 'Two new cases of rank reversals when the AHP and some of its Additive variants are used that do not occur with the multiplicative AHP', Journal MCDA, vol 10 , issue 1, pp. 11-25.

[31] Omkarprasad S. Vaidya, Sushil Kumar,(2006). Analytic hierarchy process: An overview of applications,European Journal of Operational Research 169 , pp. 1–29 ,

[32] Vidal, Ludovic-Alexandre, Sahin, Evren; Martelli, Nicolas; Berhoune, Malik; Bonam, Brigitte (2010). Applying AHP to select drugs to be produced by anticipation in a chemotherapy Compounding unit. Expert System with Applications, Vol – 37, issue 2, pp.1528 – 1534.

[33] Watson, S.R. & A.N.S. Freeling (1982). Assessing Attribute Weights. Omega 10(6), pp. 582-583.

[34] William C. Wedley, Bertram Schoner, (1989), 'Ambiguous Criteria Weights in AHP: Consequences and Solutions ' Decision Sciences, Volume 20, Issue 3, pp. 462-475.

[35] Milan Zeleny,(1980). Management Support Systems: Towards Integrated Knowledge Management, Human Systems Management, 7, (1987) pp. 59 – 70.