Three Dimensional Unsteady Flow of Blood in Arteries with Multiple Stenoses

H. Girija BaI, K.B. Naidu and G. Vasanth Kumar

Abstract--- *In this study, three dimensional unsteady blood flow in arteries with two axisymmetric stenoses of dilation of 25% has been numerically analyzed. Using computational fluid dynamics techniques, hemodynamic factors such as velocity, pressure and streamline pattern are investigated. The problem is solved by finite volume method. Numerical simulation is prescribed using the CFD software's Fluent and Gambit. Low pressure and high velocity are observed in the regions of stenoses. This is an indication to the interruption of blood flow. Detection and quantification of multiple stenoses serve as the root for surgical interference. These techniques based on computer flow study are important for understanding the relationship between hemodynamic parameters and hazard of rupture.*

Keywords--- *Multiple Stenoses, Computational Fluid Dynamics, Hemodynamics, Numerical simulation, Finite Volume Method*

I. INTRODUCTION

HEMODYNAMICS refers to physiological factors governing the flow of blood in circulatory system. Blood flow in arteries is dominated by unsteady flow phenomena .Blood flow over normal physiological situation is a vital field of study, as is blood flow under diseased circumstances. In developed countries majority of deaths are caused from cardiovascular diseases, most of which are connected with some form of irregular blood flow in arteries. The arteries are living organs that can adjust to and change with the varying hemodynamic conditions. The deposit of cholesterol and proliferation of connective tissues in an arterial wall forms plaques which grow inward of the artery and restrict the natural blood flow.

The obstruction may damage the internal cells of the wall and may lead to further growth of stenosis. Stenosis refers to the abnormal narrowing of a blood vessel due to the development of arteriosclerotic plaques or other types of abnormal tissue development. This vascular disease is of frequent occurrence, particularly in mammalian arteries. If this disease takes a severe form, it may lead to fatality. Although the exact mechanism for the development of this vascular disease is somewhat unclear, various investigators emphasisssed that the ormation of the intravascular plaques and the impingement of ligaments and spure on the blood vessel wall are some of the major factors for the initiation and development of this vascular disease.

The homodynamic behavior of the blood flow in arterial stenoses bears some important aspects due to engineering interest as well as feasible medical applications. In 1968, Young analyzed the effect of stenosis in circular tube and the dart. J. Daly (1976) [11] made a numerical study of pulsatile flow through stenosed canine femoral arteries for lumen constrictions in the range 0-61%. In 1985, V. O'Brien and L.W. Ehrlich [1] analyzed a simple pulsatile flow in an artery with a constriction. In 1995 H. Huang et. al. [9] investigated flow in a tube with an occlusion by using finite difference scheme for steady and unsteady flow. In 2006, Somkid Amornsanmankul et. al. [4] studied the pulsatile flow of blood through stenotic artery. In 2008, Seung E. Lee et.al [7] numerically analyzed the blood flow dynamics in a stenosed, subject-specific, carotid bifurcation using the spectral element method. In 2008, Quan Long et. al. [8] made a numerical study to investigate the capacity of the cirlce of Wills(CoW) to provide collateral blood supply for patients with unilateral carotid arterial stenosis. In 2010, V.P. Srivastava et. al. [3] investicated the effects of overlapping blood flow characteristics in a narrow artery.

The simulation of complex dynamic processes that appear in nature or in industrial applications poses a lot of challenging mathematical problems, opening a long road from the basic problem, to the mathematical modelling, the numerical simulation, and finally to the interpretation of results.

In general, the cycle of (mathematical) modeling consists of the following steps [9];

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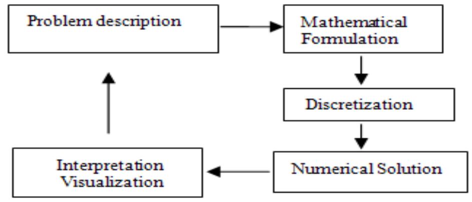

The aim of the present study is to find the unsteady flow pattern for the multiple stenoses using CFD analysis which has not been thoroughly investigated so far. Axisymmetric pulsatile flow of a viscous fluid in a constricted vessel is considered. Limitations on the amount of the constriction are ignored. Since arterial wall is gently elastic, we neglect the wall dispensability. Change in diameter in arteries is on the order of 10% [12]; so error in fixed diameter is minute.

II. METHODS

A. Formulation of the Problem

The geometry of the stenosed vessel in Fig.1. is given by

$$y(x) = \begin{cases} a\left(l - \frac{\delta}{2a}\left(l + \cos\left(\frac{\pi x}{x_0}\right)\right)\right), & |x| \le x_0 \\ a, & |x| > x_0 \end{cases} \quad (1)$$

B. Boundary Conditions

u=0, v=0 ,w = 0, 0n stenosed vessel

u=u(t), v=0, w=0,on inflow segment

f_x=0, f_y=0, f_z =0, on outflow segment

$\boldsymbol{f}_x$ and $\boldsymbol{f}_y$ are the components of an arbitrary vector valued function f defined by

$$\overline{f} = \overline{n}\ .\ \overline{\tau} = -\,p\,\overline{n} + \mu\,[\overline{n}.(\nabla\overline{u}) + \nabla.(\overline{n}.\overline{u})] \quad [2]$$

C. Governing Differential Equations

Equations of momentum and mass conservation for incompressible fluid can be written as:

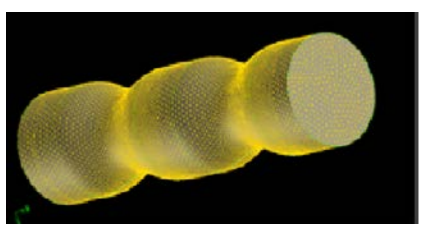

Fig.1: An Arterial Vessel Model with Two Asymmetric Stenoses

$\nabla.\overline{v} = 0$ (2)

$$\rho\left(\frac{\partial\overline{v}}{\partial t} + \overline{v}.\nabla\overline{v}\right) = -\nabla p + \mu\nabla^2\overline{v} \quad (3)$$

where: ρ - density of blood, $\overline{v}$ -velocity field, p - pressure, μ = co-efficient of viscosity.

The Navier-Stokes System of Equations: Incompressible Viscous Flow case:

The vector form of unsteady three-dimensional motion of a viscous, incompressible and isothermal flow:

$$\frac{\partial\vec{Q}}{\partial t} + \frac{\partial\vec{E_i}}{\partial x} + \frac{\partial\vec{F_i}}{\partial y} + \frac{\partial\vec{G_i}}{\partial z} = \frac{\partial\vec{E_v}}{\partial x} + \frac{\partial\vec{F_v}}{\partial y} + \frac{\partial\vec{G_v}}{\partial z} \quad [13] \quad (4)$$

Where $\vec{Q}$ is the vector containing the primitive variables and E_i, F_i , G_i are the vectors containing the inviscid fluxes in the x, y and z directions is given by

$$\vec{Q} = \begin{bmatrix} 0 \\ u \\ v \\ w \end{bmatrix}, \mathrm{E_i} = \begin{pmatrix} u \\ u^2 + p \\ uv \\ uw \end{pmatrix}, \mathrm{F_i} = \begin{pmatrix} v \\ vu \\ v^2 + p \\ vw \end{pmatrix},$$

$$\mathrm{G_i} = \begin{pmatrix} w \\ wu \\ wv \\ w^2 + p \end{pmatrix}$$

The viscous fluxes in the x, y and z directions are defined as follows

$$\mathrm{E_v} = \begin{pmatrix} 0 \\ \tau_{xx} \\ \tau_{xy} \\ \tau_{xz} \end{pmatrix} \quad \mathrm{F_v} = \begin{pmatrix} 0 \\ \tau_{xy} \\ \tau_{yy} \\ \tau_{yz} \end{pmatrix}, \quad \mathrm{G_v} = \begin{pmatrix} 0 \\ \tau_{zx} \\ \tau_{zy} \\ \tau_{zz} \end{pmatrix},$$

Since we made the assumptions of an incompressible flow, appropriate nondimensional terms and expressions for shear stresses must be used; these expressions are given as follows

$$\tau_{xx} = \frac{2}{Re_L}\frac{\partial u}{\partial x},$$

$$\tau_{yy} = \frac{2}{Re_L}\frac{\partial v}{\partial y},$$

$$\tau_{zz} = \frac{2}{Re_L}\frac{\partial w}{\partial z},$$

$$\tau_{xy} = \frac{1}{Re_L}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \tau_{yx} \ ,$$

$$\tau_{xz} = \frac{1}{Re_L}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) = \tau_{zx},$$

$$\tau_{yz} = \frac{1}{Re_L}\left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right) = \tau_{zy}$$

Transformation of the governing equations from cartesian coordinates (x,y,z,t) to computational space with generalised curvilinear coordinates (ξ, η, ς, t)

$$\frac{\partial \hat{Q}}{\partial t} + \frac{\partial \widehat{E_\iota}}{\partial \xi} + \frac{\partial \widehat{F_\iota}}{\partial \eta} + \frac{\partial \widehat{G_\iota}}{\partial \varsigma} = \frac{\partial \widehat{E_v}}{\partial \xi} + \frac{\partial \widehat{F_v}}{\partial \eta} + \frac{\partial \widehat{G_v}}{\partial \varsigma} \qquad (5)$$

Where

$$\left.\begin{aligned}
\hat{Q} &= \frac{\vec{Q}}{J_x} \\
\widehat{E_\iota} &= \frac{1}{J_x}\left(\xi_x \overrightarrow{E_\iota} + \xi_y \overrightarrow{F_\iota} + \xi_z \overrightarrow{G_\iota}\right) \\
\widehat{F_\iota} &= \frac{1}{J_x}\left(\eta_x \overrightarrow{E_\iota} + \eta_y \overrightarrow{F_\iota} + \eta_z \overrightarrow{G_\iota}\right) \\
\widehat{G_\iota} &= \frac{1}{J_x}\left(\varsigma_x \overrightarrow{E_\iota} + \varsigma_y \overrightarrow{F_\iota} + \varsigma_z \overrightarrow{G_\iota}\right) \\
\widehat{E_v} &= \frac{1}{J_x}\left(\xi_x \overrightarrow{E_v} + \xi_y \overrightarrow{F_v} + \xi_z \overrightarrow{G_v}\right) \\
\widehat{F_v} &= \frac{1}{J_x}\left(\eta_x \overrightarrow{E_v} + \eta_y \overrightarrow{F_v} + \eta_z \overrightarrow{G_v}\right) \\
\widehat{G_v} &= \frac{1}{J_x}\left(\varsigma_x \overrightarrow{E_v} + \varsigma_y \overrightarrow{F_v} + \varsigma_z \overrightarrow{G_v}\right)
\end{aligned}\right\} \qquad (6)$$

$\widehat{(Q)}$ is the vector containing the primitive variables and $\widehat{E_\iota}$, $\widehat{F_\iota}$, and $\widehat{G_\iota}$ are the vectors containing the inviscid fluxes in the ξ, η and ζ directions respectively, and are given by

$$\hat{Q} = \frac{1}{J_x}\begin{pmatrix} 0 \\ u \\ v \\ w \end{pmatrix}$$

$$\widehat{E_\iota} = \frac{1}{J_x}\begin{bmatrix} U \\ uU + p\xi_x \\ vU + p\xi_y \\ wU + p\xi_z \end{bmatrix}; \ \widehat{F_\iota} = \frac{1}{J_x}\begin{bmatrix} V \\ uV + p\eta_x \\ vV + p\eta_y \\ wV + p\eta_z \end{bmatrix}, \ \widehat{G_\iota} = \frac{1}{J_x}\begin{bmatrix} W \\ uW + p\varsigma_x \\ vW + p\varsigma_y \\ wW + p\varsigma_z \end{bmatrix} \text{[13]} \qquad (7)$$

Where U, V and W are the contravariant velocities. The shear stresses in the computational space C are obtained and substituting in viscous flux vectors $\widehat{E_v}$, $\widehat{F_v}$ and $\widehat{G_v}$ in the ξ, η and ζ directions respectively.

$$\widehat{E_v} = \frac{1}{J_x Re_L}\begin{bmatrix} 0 \\ (\nabla\xi \,.\nabla\xi)u_\xi + (\nabla\xi \,.\nabla\,\eta)u_\eta + (\nabla\xi \,.\nabla\zeta)u_\zeta \\ (\nabla\xi \,.\nabla\xi)v_\xi + (\nabla\xi \,.\nabla\,\eta)v_\eta + (\nabla\xi \,.\nabla\zeta)v_\zeta \\ (\nabla\xi \,.\nabla\xi)w_\xi + (\nabla\xi \,.\nabla\,\eta)w_\eta + (\nabla\xi \,.\nabla\zeta)w_\zeta \end{bmatrix}$$

Similarly to calculate $\widehat{F_v}$ and $\widehat{G_v}$

(8)

Equations (5), (6), (7) and (8) are the governing equations of an incompressible viscous flow in strong conservation form in computational space. These equations has to be solved using difference method.[13]

D. *Methodology*

Computational fluid dynamics is used in many areas, such as engineering and medical field. This new field provides very detailed information about fluid characteristics. Medical science lent this new technology to study hemodynamic within the body. CFD uses Finite Volume Method (FVM) as like FEM, FDM. In FVM ,two interpolation structures can be used: (i) Piecewise constant interpolation (ii) Piecewise linear (or bilinear) interpolation. Ist is denoted by cell-centered method and the second is denoted by the cell-vertex method. In both methods, the cells and group of cells around a node are used as volumes.

A CFD solution involves the following basic steps:

- Creation of the geometry
- Choice of the models
- Apply of the boundary conditions
- Flow field computation
- Post processing

CFD helps us to understand their formation, growth, and rupture of stenosis. The purpose of our study is to show the possibility of development of computational analyses of velocity, pressure, and patterns of streamlines.

Discretization is conducted in Gambit. Governing equations were solved in FLUENT which use finite volume method. The file obtained in Gambit is imported in software program FLUENT v.6.3.26 (ANSYS. Inc.) and processed in 3 stages, namely preprocessing, processing and post processing.

Although blood has actually non- Newtonian behavior, in the simulation it is considered Newtonian because there were no significant differences in the distribution of wall shear stress [6].

Reynolds number and Womersley number are the two physical parameters necessary to solve an incompressible fluid flow problem.

Womersley number (α) depends on: flow rate, model geometry and fluid viscosity and varies with vessel diameter.

Womersley number is: $\alpha = r\sqrt{\frac{2\pi\upsilon\rho}{\mu}}$ (6)

where: **r** [m] - entry of the vessel radius, **υ** - flow rate, **ρ** [kg/m^3] - blood density, ***μ*** [kg / ms] - blood viscosity.

The Womersley parameter α can be interpreted as the ratio of the unsteady forces to the viscous forces. When the Womersley parameter is low, viscous forces dominate, velocity profiles are parabolic in shape, and the centerline velocity oscillates in phase with the driving pressure gradient (Womersley1955, McDonald1974). For Womersley parameters above 10, the unsteady inertial forces dominate, and the flow is essentially one of piston-like motion with a flat velocity profile.[10]

The dimensionless Reynolds number (Re) gives the flow command. This number varies with the diameter of the vessel for each case. $Re = \frac{inertia\ force}{friction\ force}$

i.e. $Re = \frac{\rho v d}{\mu}$ (7)

where: **ρ** [kg/m^3] - blood density, v [m/s] - maximum speed of blood flow at the entrance, **d** [m] - diameter at the entrance of the vessel, ***μ*** [Kg/ms]-blood viscosity. [5]

Depending on this number, blood flow values in the model can be:

- Laminar when Re ≤ 2300,

- Transient when 2300 <Re <10000,
- Turbulent when Re> 10000.

If flow is laminar, wall shear stress is defined as the velocity gradient at the wall, through the relation: $\tau_\omega = \mu \frac{\partial v}{\partial n}$ (8)

where τ_ω [Pa] - tension tangential to the wall, **μ**[kg / ms] - blood viscosity, v [m / s] - the speed of blood flow in the vessel, **n** - normal direction to the vessel wall. [5]

III. RESULTS

Governing equations were solved in FLUENT which use finite volume method for discretization conducted in Gambit. Detailed study of velocity, pressure and streamlines are made.

By considering the Domain

Table 1: Domain of the Geometry

	MIN.(m)	MAX(m)
X	-2	5
Y	-1	1
Z	-1	1

Geometry of the vessel has been created using MATLAB. Imported the values in Gambit by considering blood density ρ = 1060 [kg/m^3] and dynamic viscosity **η** = 0.003 [kg/ms] (Poiseuille) and by giving boundary condition with velocity 0.6(m/s), velocity inlet, axisymmetric flow, as Interval size 0.1 grid has been generated.

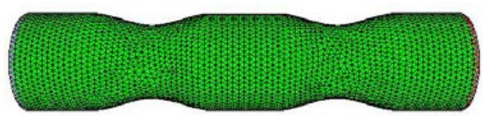

Fig.2: Grid Display for an Unsteady Flow

Table 2: By Considering Interval Size = 0.1,

Nodes	25,581
Mixed wall faces	10,040
Mixed outflow faces	752
Mixed velocity– inlet faces	752
Mixed interior faces	2,65,276
Tetrahedral cells	1,35,524

For unsteady flow for 100 time steps, 10 seconds for 1000 flow time, iterations per time step 20 and reporting interval as 10 has been calculated.

Table 3: Volume Statistics

MIN.(m^3)	MAX(m^3)
2.381751-005	3.956449e-004

Total volume = 1.90732e001

Face area statistics:

Min. face area (m^2) = 1.243686e-003

Max. face area (m^2) =1.257079e-002

The velocity magnitude for unsteady flow with inlet velocity= 0.6(m/s)

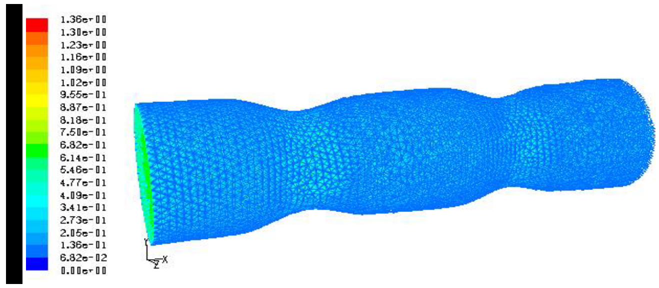

Fig.3 (a): Velocity Magnitude for an Unsteady Flow

By considering different planes ,we analyse the velocity variation prominently. At the region of stenoses velocity is high compared to other region fig 3(b)

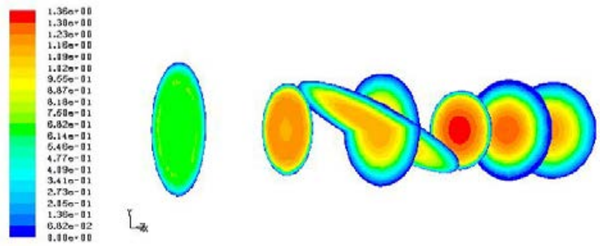

Fig.3 (b): Velocity Magnitude at Planes x= -0.0015, 0, 0.0015, 0.003, 0.004, 0.005

By considering the velocity contours for the plane section along xy- axis, setting z = 0, contours of the flow pattern has been analyzed. This contours shows that there is a circulation of flow in the region of stenoses and also the flow is getting disturbed due to contriction fig 3(c).

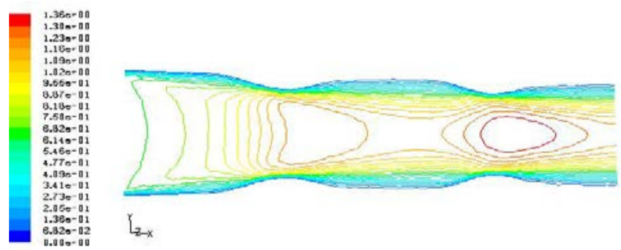

Fig.3 (c): Velocity Magnitude for a Plane Section x= -.002 to 0.005 and y = -0.001 to 0.001 and z=0

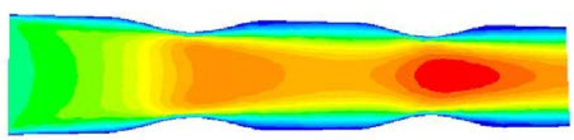

Fig.3 (d): Velocity Magnitude for a Plane Section x= -.002 to 0.005 and y = -0.001 to 0.001 and z=0, filled

Velocity drop can be analyse through the xy plot.

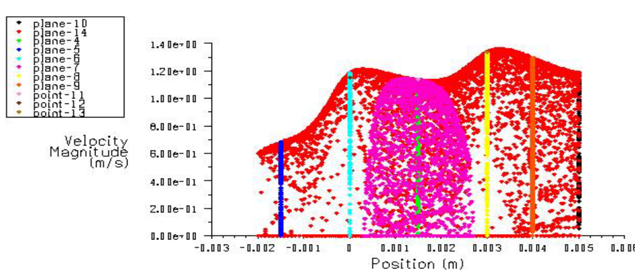

Fig.3(e): Velocity Magnitude Graph for an Unsteady Flow at Different Planes of x

From fig.3(b), 3(c), 3(d), 3(e), we observe that there is high velocity at both the region of constriction. The velocity profile is not same throughout the domain. There is a variation of velocities throughout the vessel especially due to constriction the flow is much affected and there is a restriction of flow.

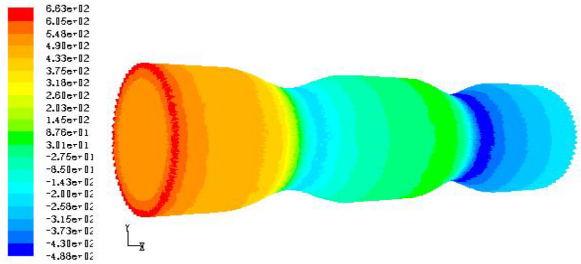

Fig. 4(a): Pressure Magnitude for an Unsteady Flow

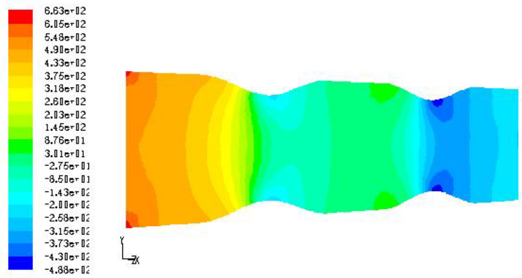

Fig. 4(b): Pressure Magnitude for a Plane Section x= -.002 to 0.005 and y = -0.001 to 0.001 and z=0

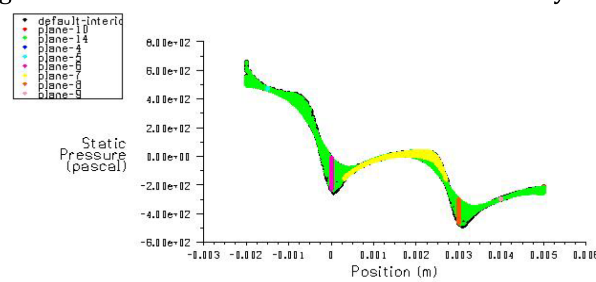

Fig. 4(c): Pressure Difference for an Unsteady Flow

It is clear that there is a considerable pressure variation in the direction normal to the X-axis in the stenosed region. A knowledge of the axial pressure loss across the stenoses relative to the overall pressure loss across the entire stenosed vessel is of great physiological interest. The pressure gradient from the inflow segment to the peak of stenoses is much higher than the pressure gradient across the whole of the stenoses and it increases with δ.

Streamlines are a family of curves that are instantaneously tangent to the velocity vector of the flow. This show the direction of fluid element which travels at any point at any time. These stream lines are representative of the speed in a given time

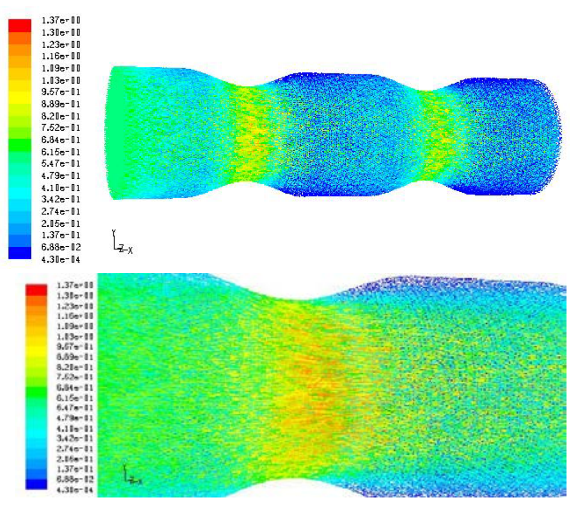

Fig. 5: Streamlines for the Fluid Particles

Wall fluxes, wall shear stress on different positions

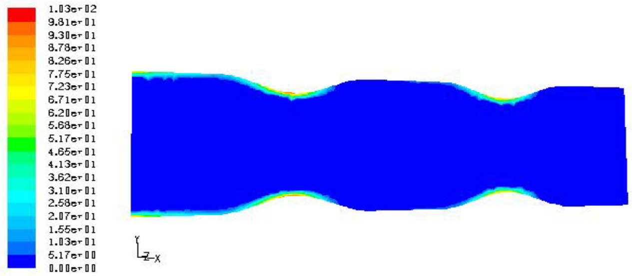

Fig. 6(a): Wall Shear Stress for an Unsteady Flow

"Flux Report"
Mass Flow Rate (kg/s)
------------ --------------------
default-interior 0.0040111574
if 0.0019948452
of -0.0019948441
wall 0
---------------- --------------------
Net 1.1641532e-09

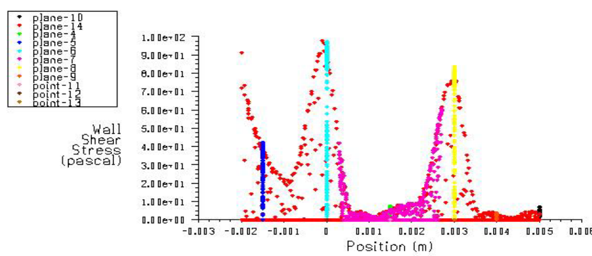

Fig. 6(b): Wall Shear Stress Plot on the Domain

Pathlines of fluid particles at different position of x is shown in fig. 7

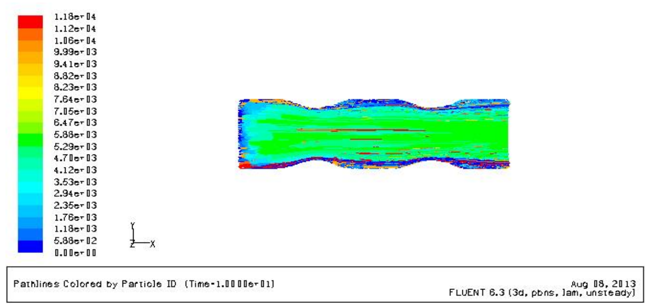

Fig. 7: Pathlines for an Unsteady Flow

Iterations were calculated and solution converges correct to the decimals with equation of continuity (black), and momemtum equation x - velocity (red), y – velocity (green), z-velocity (blue)

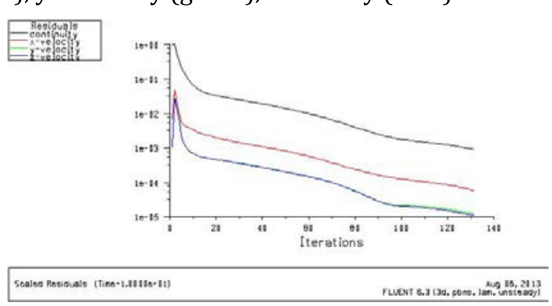

Fig. 8: Convergence of Iterations for an Unsteady Flow

In case of non-convergence, some parameters have to be tuned adequately. For explicit solvers the 'cfl' no. and for implicit solvers the under relaxation factors can be changed. The solution converges at 128th iterations where the continuity, x, y, z velocity values are -9.9286e-04, 6.5562e-05, 1.3315e-05, 1.1970e-05 respectively. The study of unsteady flow reveals several interesting new features. It appears that there is a correlation between regions of recirculation, which is a prominent feature of the unsteady flow, and the location of lesions.[9]

IV. CONCLUSION

Flow analysis shows that an unsteady 3D flow model is not similar for all stenoses. Flow characteristics are highly dependent on the geometry of the vessels and different constriction of sizes of stenoses. Low pressure and high velocity are obtained in the region of stenoses. These techniques based on computer flow study are important for understanding the relationship between hemodynamic parameters and risk of rupture. The formation of a stenoses is the most dreadful biological reaction. Hence, complete understanding of the relationship between pressure, blood flow and symptoms for cardiovascular stenoses remains a critical problem. New devices to repair stenotic arteries are now being developed. In the future, the study of arterial blood flow will lead to the prediction of individual hemodynamic flows in any patient, the development of diagnostic tools to quantify disease, and the design of devices that mimic or alter blood flow. This field is rich with challenging problems in fluid mechanics involving three-dimensional, pulsatile flows.

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