Job Sequencing Problem Using Advanced Heuristics Techniques

Pervaiz Iqbal, Dr.P.S. Sheik Uduman and Dr.S. Srinivasan

Abstract--- *Job sequencing technique is used to determine an optimal sequence. In this paper, heuristic techniques called row sum methods to obtain a sequence of jobs for solving job-sequencing problems is proposed, in order to minimize the total elapsed time of the sequence. This technique is first applied for 2-machines n-jobs problems (Model I) and extended the same for 3-machines n-jobs problems (Model II). Finally, it is extended for m-machines n-jobs problems after converting into 2-machines n-jobs problems (Model III) by using condition of Johnson's method.*

Keywords--- *Row Sum Methods, Total Elapsed Time, Optimal Sequence, Processing Time, Idle Time, etc*

I. INTRODUCTION

SEQUENCING problems have been most commonly encountered problem in real world where different products are to be processed over various combinations of machines. It is the selection of an appropriate order for a number of different tasks to be performed on a finite number of service facilities in order to make effective use of available facilities and achieve greater output.

The problem of sequencing may have some restriction placed on it, such as time for each job, order of processing of each job on each machines, availability of resources (men, machines, material and space) etc.

Number of research articles deals with job sequencing problem to frame an optimal job sequence under various circumstances. Christos Koulamas, et al. [1] studied the single job lot streaming problem in a two stage hybrid flow-shop that has m-identical machine at first stage and one machine at the second stage to minimize the make span. Kalczynski.P.J., et al. [2] deals with the classical problem of minimizing the make span in a two machine flow shop. The optimal job sequencing rule can be determined by applying Johnson's rule when the job processing time are deterministic. When they are exponent and random variables,

Talwar's rule yields a job sequence that minimizes the make span stochastically. Assuming that the job processing times are independently distributed random variables, a new job sequencing rule is presented that includes both Johnson's and Talwar's rules as special cases. Zhao. Y.F., et al. [3] addresses a problem of continuous batch scheduling problem, arising in the heating process of blooms in steel industry. Pan.Y., et al.[4] presented a simple elegant algorithm for finding an optimal solution to a general min-max sequencing problem. George Steiner, et al. [5] considers the single machine bi-criterion scheduling problem of enumerating pare to optimal sequences with respect to the total weighted completion time and the maximum lateness objectives. Drobouchevitch.I.G., et al. [6] the problem of sequencing n-jobs in a two machine re-entrant shop with the objective of minimizing the maximum completion time is considered. Cai. X., et al. [7] is concerned with the problem of scheduling n-jobs with a common due dates on a single machine so as to minimize total cost arising from earliness and tardiness. Vachajitpan. P., [8] gives computer program for solving n-job m-machine problems and it is equivalent for finding the shortest path between two nodes in finite network.

This new method is very simple and easy to understand for large job sequencing problem as compared to other methods such as Simulated Annealing, Tabu Search, Ant Colony Algorithm and Genetic Algorithm for minimizing the total elapsed of the sequence. A numerical illustration is given to clarify the algorithm.

1.1 Sequencing Problem

Sequencing problems is defined as follows, let there be n-job each of which is processed through m-machines. The order of the machines in which each job should be performed is given. And the actual and expected time required by the jobs on each of the machine is also given. The general sequencing problem is to find the sequence out of $(n!)^m$ possible sequences which minimizes the total elapsed time between the start of the first job on first machine and the completion of the last job on last machine.

II. ASSUMPTIONS

- The processing time of jobs on different machines are exactly known and are independent of the order of the jobs in which they are to be processed.
- The time taken by the job in moving from one machine to another is negligible.
- Once a job has begun on a machine, it must be completed before another job can begin on the same machine.
- Only one job can be processed on a given machine at a time.
- Machines to be used are of different types.

The order of completion of jobs is independent of the sequence of jobs.

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III. Basic Model (Processing of n-Jobs through 2-Machines)

Let there be n-jobs, each of which is processed through 2-machines, say M_1 and M_2 in the order M_1M_2. If the machine M_2 is not free at a moment for processing the same then job has to wait in a waiting line for its turn on machine M_2. Let T_{ij} (i=1,2,.......n) be the time required for processing i^{th} job on j^{th} machine. Since passing is not allowed, therefore machine M_1 will remain busy in processing all the n-jobs one by one while machine M_2 may remain idle after completion of one job and before starting of another job. The sequencing problem can be written as follows:

Jobs/Machines	J_1	J_2	....	J_n
M_1	T_{11}	T_{12}	....	T_{1n}
M_2	T_{21}	T_{22}	....	T_{2n}

Let T_{ij} is the time duration taken by i^{th} job on j^{th} machine. The objective is to minimize the idle time of the second machine. Let X_{2j} be the time for which machine M_2 remain idle after finishing $(j-1)^{th}$ job and before start processing the j^{th} job (j=1,2,......,n). The total elapsed time T is given by

$$T = \sum_{j=1}^{n} T_{2j} + \sum_{j=1}^{n} X_{2j}$$

Where X_{2j}'s may or may not be zeros. The problem is to find the sequence for n-jobs on 2-machine which will give minimized total elapsed time.

IV. Model-I (Processing of n-Jobs Through 2-Machines using Advance Heuristic Method)

4.1 Row Sum Method for Processing N-Jobs through 2-Machines

We now define the following terms which will be used in the proposed new method called row sum method for determining an optimal sequence of jobs in job sequencing problems. Let R_{1i} and R_{2i} is the time duration of jobs (j=1,2,.....,n) on machines M_1 and M_2. Take the sum of the given two rows as $\chi = \sum_{i=1}^{n}(R_{1i} + R_{2i})$, where and select the minimum value. After selecting minimum value from the sum, check the corresponding minimum time of jobs on machines M_1 and M_2. If the processing time is minimum on machine M_1, then the corresponding job is placed in first position of the sequence. If it is minimum for machine M_2, then place the corresponding job in last position of the sequence.

4.2 Algorithm

1. Start.
2. Read the number of jobs and processing times for each jobs on machine M_1 and M_2.
3. Add the processing time of machines M_1 and M_2, as $\chi = \sum_{i=1}^{n}(R_{1i} + R_{2i})$, where R_{1i} and R_{2i} is the time duration of jobs (j=1,2,.....,n) on machine M_1 and M_2.
4. From the sum, find the smallest sum. If the smallest processing time is for the first machine M_1, then place the corresponding job in the first available position in the sequence.
5. If it is for the second machine, then place the corresponding job in the last available position in the sequence.
6. If there is a tie in selecting the minimum processing time, then there may be three situations.
   - Minimum among all processing times is the same for the machine, min $(T_{1j}, T_{2j}) = T_{1k} = T_{2r}$, then process the k^{th} job first and r^{th} job last.
   - If the tie occurs among processing times T_{1j} on machine M_1 only, then select job corresponding to the smallest job subscript first.
   - If the tie occurs among processing times T_{2j} on machine M_2, then select job corresponding to the largest job subscript last.
7. Delete the particular job from the next consideration.
8. Repeat the above procedure and check all jobs are assigned a position in the optimal sequence.
9. Calculate the overall elapsed time
10. Calculate the idle time for machines M_1 and M_2.
11. End.

4.3 Problems Involving Processing n-Jobs through 2-Machines

Example: Determine the optimal sequence of jobs that minimizes the total elapsed time required to complete the following jobs on machines M_1 and M_2 in the order M_1M_2.

Jobs/Machines	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
M_1	2	5	4	9	6	8	7	5	4
M_2	6	8	7	4	3	9	3	8	11

Solution: The sum of the rows of the given table for each job on machines M_1 and M_2 is:

Jobs/Machines	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
Sum	8	13	11	13	9	17	10	13	15

The job sequence for the above table is:

J_1								

The reduced table for remaining job sequence is:

Jobs/Machines	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
M_1	5	4	9	6	8	7	5	4
M_2	8	7	4	3	9	3	8	11

The sum of the rows of the reduced table for each job on machines M_1 and M_2 is:

Jobs/Machines	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9
Sum	13	11	13	9	17	10	13	15

The job sequence for the reduced table is:

J_1								J_5

The reduced table for remaining job sequence is:

Jobs/Machines	J_2	J_3	J_4	J_6	J_7	J_8	J_9
M_1	5	4	9	8	7	5	4
M_2	8	7	4	9	3	8	11

The sum of the rows of the reduced table for each job on machines M_1 and M_2 is:

Jobs/Machines	J_2	J_3	J_4	J_6	J_7	J_8	J_9
Sum	13	11	13	17	10	13	15

The job sequence for the reduced table is:

J_1							J_7	J_5

The reduced table for remaining job sequence is:

Jobs/Machines	J_2	J_3	J_4	J_6	J_8	J_9
M_1	5	4	9	8	5	4
M_2	8	7	4	9	8	11

The sum of the rows of the reduced table for each job on machines M_1 and M_2 is:

Jobs/Machines	J_2	J_3	J_4	J_6	J_8	J_9
Sum	13	11	13	17	13	15

The job sequence for the reduced table is:

J_1	J_3	J_2	J_8			J_4	J_7	J_5

The reduced table for remaining job sequence is:

Jobs/Machines	J_6	J_9
M_1	8	4
M_2	9	11

The sum of the rows of the reduced table for each job on machines M_1 and M_2 is:

Jobs/Machines	J_6	J_9
Sum	17	15

The required job sequence for the given table is:

J_1	J_3	J_2	J_8	J_9	J_6	J_4	J_7	J_5

Total Elapsed Time

Jobs sequence /Machines	Machine-M_1		Machines-M_2	
	Time In	Time Out	Time In	Time Out
J_1	0	2	2	8
J_3	2	6	8	15
J_2	6	11	15	23
J_8	11	16	23	31
J_9	16	20	31	42
J_6	20	28	42	51
J_4	28	37	51	55
J_7	37	44	55	58
J_5	44	50	58	61

Therefore the total elapsed time is 61 hrs. And the idle time on machines M_1 and M_2 are 11 hrs, and 2hrs respectively.

V. Model II (Processing of n-jobs through 3-machines)

Let there be n-jobs each of which is to be processed through 3-machines, M_1, M_2 and

M_3 in the order $M_1M_2M_3$. Let T_{ij} (i=1,2,3 ; j=1,2,......n) be the time required for processing i^{th} job on j^{th} machine. The time duration is follows.

Jobs/Machines	J_1	J_2	....	J_n
M_1	T_{11}	T_{12}	....	T_{1n}
M_2	T_{21}	T_{22}	....	T_{2n}
M_3	T_{31}	T_{32}	....	T_{3n}

5.1 Row Sum Method for Processing n-Jobs through 3-Machines

Convert the n-jobs 3-machine problems into n-jobs 2-machine problems by checking either of the conditions.

1. Minimum of $M_1 \geq$ Maximum of M_2
2. Minimum of $M_3 \geq$ Maximum of M_2

Or both hold.

5.2 Algorithm

1. Examine processing times of given jobs on all the 3-machines and if either of the above conditions hold, then go to step-2, otherwise the algorithm fails.
2. Introduce two fictitious machines, G and H with corresponding processing times given by

$$T_{Gj} = T_{1j} + T_{2j},\ j = 1,2,.....,n$$

That is, the processing time on machine G is the sum of the processing times on machines M_1 and M_2.

$$T_{Hj} = T_{2j} + T_{3j},\ j = 1,2,......,n$$

That is, the processing time on machine H is the sum of the processing times on machines M_2 and M_3.

3. Determine the optimal sequence of jobs for the n-jobs, 2-machine equivalent sequencing problem with the prescribed ordering GH in the same way as discussed earlier. Applying the row sum method for the reduced n-jobs 2-machines job sequencing problem obtained from above conditions and we get the required job sequence.

5.3 Problems involving processing n-Jobs through 3-Machines

Example: Determine the sequence that minimizes the total elapsed time required to complete the following tasks on the machines in the order $M_1M_2M_3$. Find also minimum total elapsed time and the idle time on the machines.

Jobs/Machines	J_1	J_2	J_3	J_4	J_5	J_6	J_7
M_1	3	8	7	4	9	8	7
M_2	4	3	2	5	1	4	3
M_3	6	7	5	11	5	6	12

Solution: Convert the 3-machines problems in to 2-machines problem. Here min (t_{M1j}) = 3; min (t_{M3j}) = 5; max (t_{M2j}) = 5. Since min (t_{M3j}) $\geq$ max (t_{M2j}) for all j is satisfied, the given problem can be converted into a 7-jobs and 2-machines. The processing time on two fictitious machines G and H can be determined by the following relationships:

$$T_{Gj} = T_{m1j} + T_{m2j}$$

and

$$T_{Hj} = T_{m2j} + T_{m3j}$$

The processing times for the new problem are given below:

Jobs/Machines	J_1	J_2	J_3	J_4	J_5	J_6	J_7
G	7	11	9	9	10	12	10
H	10	10	7	16	6	10	15

The sum of the rows of the given table for each job on machines M_1 and M_2 is follows:

Jobs/Machines	J_1	J_2	J_3	J_4	J_5	J_6	J_7
Sum	17	21	16	25	16	22	25

The job sequence for the above table is:

J_1	J_4	J_7	J_6	J_2	J_5	J_3

Total Elapsed Time

Jobs sequence /Machines	Machine-M_1		Machine-M_2		Machines-M_3	
	Time In	Time Out	Time In	Time Out	Time In	Time Out
J_1	0	3	3	7	7	13
J_4	3	7	7	12	13	24
J_7	7	14	14	17	24	36
J_6	14	22	22	26	36	42
J_2	22	30	30	33	42	49
J_5	30	39	39	40	49	54
J_3	39	46	46	48	54	59

Therefore the total elapsed time is 59 hrs. And the idle times on machines M_1, M_2 and M_3 are 13 hrs, 37 hrs and 7hrs respectively.

VI. Model III: (Processing of n-jobs through m-machines)

Jobs/Machines	J_1	J_2	....	J_n
M_1	T_{11}	T_{12}	....	T_{1n}
M_2	T_{21}	T_{22}	....	T_{2n}
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
M_m	T_{m1}	T_{m2}	....	T_{mn}

Convert the n-jobs 3-machine problems into n-jobs 2-machine problems by checking either of the conditions.

- Minimum of $M_1 \geq$ Maximum of (M_2,M_3,........,M_{m-1})
- Minimum of $M_m \geq$ Maximum of (M_2,M_3,........,M_{m-1})

Or both hold and by using the procedure already discussed above.

Again applying the same for the reduced n-jobs two 2-machines job sequencing problem obtained from above conditions and get the required job sequence.

VII. Conclusion

In this paper, a new method called row sum method is introduced, to frame a sequence of n-jobs 2-machines and n-jobs 3-machines for solving the job sequencing problem, and also it satisfy for n-jobs m-machines problem. In near future this new method can be extended to determine a sequence of n-jobs through 2 and more than 2-machines without using existing conditions.

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